INTERFERENCE TESTING USING HORIZONTAL WELLS IN ANISOTROPIC RESERVOIRS

BY

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THESIS ABSTRACT

Name: AWOTUNDE ABEEB ADEBOWALE

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This thesis presents a model for interference testing utilizing two horizontal wells of unequal lengths. Interference testing between two finite conductivity horizontal wells has not been fully addressed in the literature. The purpose of this work is to present analytical pressure-transient solution for interference testing utilizing two finite-conductivity horizontal wells having unequal lengths.

This work extends the previous interference testing model between two equal-length horizontal wells to case in which the two horizontal interfering wells are of unequal length. A sensitivity study is performed to show the effect of well length on

1. Flux distribution in both wells.
2. Pressure response in both wells.
3. Appearance of flow regimes in both wells.

The new model has been validated using two previous interference testing models.

ملخص الرسالة

اسم الطالب : اوتوند ابيب اديبويل.
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هذاالبحث يقدم نموذج رياضي للضغط العابر للفحص التداخلي باستخدام بئرين افقيين مختلفين في . الفحص التداخلي بين بئرين افقيين محدودي التوصيلية تم دراسته بشكل كبير في الادبيات . ان هدف هذا العمل هو تقديم حل رياضي تحليلي للضغط العابر للفحص التداخلي باستخدام بئرين افقيين مختلفين.
هذا العمل يمدد نمذج الفحص التداخلي السابق باستحدام بئرين متساويان في الطول للحالة التي يكون فيها البئرين غير متساويين في الطول.
دراسة دقيقة اجريت لاظهار تاثير طول البئر على:

1- توزيع الجريان في كلا البئرين.
2- استجابة الضغط في كلا البئرين.
3- ظهور انظمة الجريان في كلا البئرين.

النموذج الجديد تم تاكيد صحته باستخدام نموذجي فحص الابار السابقة الذكر.

CHAPTER 1

INTRODUCTION

Interference tests utilizing a pair of horizontal wells in a reservoir provide more valuable information about the reservoir than those utilizing vertical wells. Many dimensionless solutions have been presented for interference testing of horizontal wells. However, most of these solutions assume that the horizontal observation well is not influenced by the wellbore dynamics. Recently, Al-Khamis, Ozkan and Raghavan [17, 18] derived solutions for interference tests between horizontal wells incorporating the effects of wellbore hydraulics in both wells. Their model closely resembles the actual scenario expected in a wellbore-reservoir system. The model developed by Al-Khamis et al [17, 18] however, assumes that the interfering wells are of same length. The aim of this work is to extend Al-Khamis et al [18]'s model to handle the case when both horizontal wells are of unequal lengths.

1.1 Statement of Problem

This work seeks to extend the semi-analytical solutions [17, 18] for interference testing to take care of two horizontal wells of unequal lengths. We assume an anisotropic but homogeneous reservoir in which the flow is single-phase and isothermal. We also consider a fluid of constant compressibility and viscosity.

The model presents a system in which there is non-uniform flux distribution in both the active and observation horizontal wells. Due to the high conductivity of the observation well relative to the reservoir, the observation well serves as a flow path for reservoir fluids moving towards the active well. Thus, fluid is expected to enter the observation well from its far end and move towards the near end where it exits before approaching the active well.

1.2 OBJECTIVES

The objective of this work is to:

- Extend the semi-analytic model developed by Al-Khamis, Ozkan and Raghavan [17, 18], for interference testing between a pair of horizontal wells to handle cases in which both wells are of unequal lengths.
- Study the effect of varying well lengths on pressure response in both wells.

1.3 APPROACH

This work will adopt the approach developed by Al-Khamis et al [17, 18] in coupling the reservoir and wellbore flow models. Because non-uniform flux distribution is assumed in the horizontal wells, the fluxes are unknown and are therefore obtained using the Newton-Raphson iterative technique.

To obtain the wellbore flow model, a steady-state momentum equation for single phase isothermal flow in a horizontal wellbore was used:

1.4 Significance of This Study

The work of Al-Khamis et al. [17, 18] is very useful in evaluating interference testing between horizontal wells in anisotropic reservoirs. However, the assumption of equal well length made in their work makes the model not applicable to many practical situations. This work has gone a step further to develop a model that could be used for interference testing between horizontal wells of unequal lengths.

CHAPTER 2

Literature Review

Extensive work has been done on single horizontal well testing but only a few is available on interference testing using a set of horizontal wells. Analytical solutions for the pressure behavior of uniform flux as well as infinite-conductivity horizontal wells have been discussed in the literature.

Clonts and Ramey (1986) [5] considered the pressure response of a uniform-flux horizontal drainhole in an anisotropic reservoir of uniform thickness, but infinite horizontal extension. They identified two possible transient flow regimes. The first is that characterized by an initial radial flow perpendicular to the drainhole axis followed by a transition to a pseudo-radial flow period. In the second transient flow behavior, the initial flow period ends simultaneously and flow is then characterized by early time linear flow followed by a transition to late time pseudo-radial flow.

Using successive integral transforms, Goode and Thambynayagam (1987) [2] presented a solution for the infinite-conductivity horizontal well located in a semi-infinite, homogeneous and anisotropic reservoir of uniform thickness and width.

Daviau et al. (1988) [4] also analyzed the pressure behavior of horizontal wells, considering both infinite-conductivity and uniform-flux inner boundary conditions. They noted that the infinite conductivity approximation related more closely to the real case than the uniform flux approximation.

Kuchuk et al. (1988) [6] extended the previous works on pressure transient behavior of horizontal wells to include the effect of gas cap and/or aquifer. They computed the pressure response at the well by averaging the pressure along the length of the well, rather than using an equivalent pressure point.

The solution presented by Babu and Odeh (1988) [8] is based on the uniform-flux condition at the wellbore. From a limited sensitivity study of numerical results from their solution, Babu and Odeh [8] argued that the uniform-flux pressure assumption resulted in approximately uniform pressure along the well length. The uniform-flux solution [8] has also been used by Babu and Odeh (1989) [9] in a study on the productivity of horizontal wells.

Ozkan et al. (1989) [3] compared the performances of horizontal wells and fully penetrating vertical fractures. For the horizontal wellbore, both infinite conductivity and uniform-flux boundary conditions were used. Ozkan et al. [3] concluded that for two horizontal drainholes drilled in diametrically opposite directions from a single vertical well, either the infinite-conductivity or the uniform-flux assumption was appropriate.

Babu and Odeh (1990) [7] noted that the infinite or semi-infinite assumption of the reservoir in the horizontal plane, used by previous authors, could lead to the occurrence or non-occurrence of some of the transient flow regimes. Therefore, they assumed the reservoir to be completely sealed in all three directions, and identified four possible transient flow regimes for a horizontal well in a closed, box-shaped reservoir. Details of the solution used by Babu and Odeh (1990) [7] appears in an earlier paper [8].

Malekzadeh and Tiab (1991) [15] studied the interference testing of horizontal-vertical wells using an analytical model based on the method of source functions. Their model assumes permeability isotropy in the horizontal plane. Using analytical solution of Clonts and Ramey [5], they presented dimensionless pressure and pressure derivative type curves for interference testing of horizontal wells and equations to estimate transmissibility and storativity from interference test data.

Zhu and Tiab (1991) [22] have presented analytical solution for multipoint interference testing in a single horizontal well located in an infinite reservoir with a linear discontinuity. Transient pressure data are measured at one or more perforated horizontal sections, while fluid is produced at alternate sections. Analysis of the data may yield estimates of permeability, degree of communication between adjacent regions, and the location and number of reservoir boundaries.

Malekzadeh (1992) [16] presented a solution for interference testing between horizontal and vertical wells and developed a correlation between the observation well responses and the exponential integral solution. He concludes that the early-time radial and linear flow periods are not observed in multi-well interference testing of horizontal wells.

Kuchuk and Habashy (1996) [24] presented new analytic solutions for horizontal wells with wellbore storage and skin in layered reservoirs with cross-flow. In their model, the layered system may be bounded by two horizontal impermeable bounding planes at the top and bottom, or either one of the bounding planes can have constant pressure, while the other is maintained as a no flow boundary. In deriving the new solution, they utilize the reflection and transmission concepts of electromagnetism to solve fluid flow problems in three dimensions.

Kuchuk and Habashy (1997) [25] further presented a general method of solving pressure transient equation in laterally composite reservoirs using an approach similar to that utilized for the layered reservoir. The model assumes that heterogeneity is in only one direction and a general Green's function for a point source in 3D laterally composite reservoir is developed by using the reflection and transmission method.

Issaka and Ambastha (1997) [23] presented a study that focuses on analyzing pressure transient data for a horizontal well with pressure recorders placed at various locations along the wellbore or away from the wellbore. They presented two methods for estimating horizontal well length using the analytical solution presented by Babu and Odeh (1990) [7] and concluded that the accuracy of estimates of horizontal well length from single-well interference tests depends on the location of the observation point (i.e. the pressure recorder).

Alkhonifer and Ershagi (1999) [20] presented a method to detect channel sands and vertical shale continuity using interference responses of parallel horizontal wells. Their method is based on integrating the responses at multiple isolated probing points along a horizontal observation well path, and mapping the permeability profiles from the application of a hybrid method that consists of deterministic and stochastic models.

Al-Khamis, Ozkan and Raghavan (2001) [17] presented their study on interference testing with horizontal observation wells in which they discussed the flow regimes in the observation well and the effect of anisotropy on interference between two horizontal wells. They developed a semi-analytical model based on the superposition of two finite-conductivity horizontal well solutions. Their development of the finite-conductivity horizontal well solution involves coupling of wellbore and reservoir flow equations as discussed by Ozkan et al. [10, 11 and 12].

Al-Khamis, Ozkan and Raghavan (2003) [18 and 19] developed semi-analytical models for interference tests of parallel and non-parallel horizontal wells. The models were used to investigate the general characteristics of interference test responses in horizontal wells. Their results indicated that the vertical interference well assumption is not valid for interference tests between two horizontal wells.

Houali and Tiab (2004) [21] also carried out a study on interference testing of horizontal wells in an anisotropic medium. Here, they presented the type curves of dimensionless pressure and pressure derivative, and the corresponding equations to calculate the directional permeabilities in the horizontal plane and the storativity between the interfering wells.

Al-Anazi and Ershaghi (2004) [20] presented a new approach for testing and analyzing pressure interference between horizontal wells for mapping the heterogeneity of inter-well flow systems. Their formulation uses Kelvin point source solution to model the performance of various point source recorders on the observation horizontal well. Their model assumes an isotropic homogenous reservoir in an infinite slab bounded by two no-flow boundaries in the vertical direction. They validated their analytical pressure solution by numerical solution.

Chapter 3

Model Development

This chapter deals with the development of the model that can adequately describe the interference test between two horizontal wells of unequal lengths. Here, we discuss the physical model and subsequently develop the semi-analytical model that describes the physical model.

3.1 Physical Model

We assume an anisotropic but homogeneous reservoir of uniform thickness h. We also assume an isothermal and a single-phase flow of fluid of constant compressibility and viscosity. The initial reservoir pressure is assumed uniform. The physical model is shown in Figure 3.1.

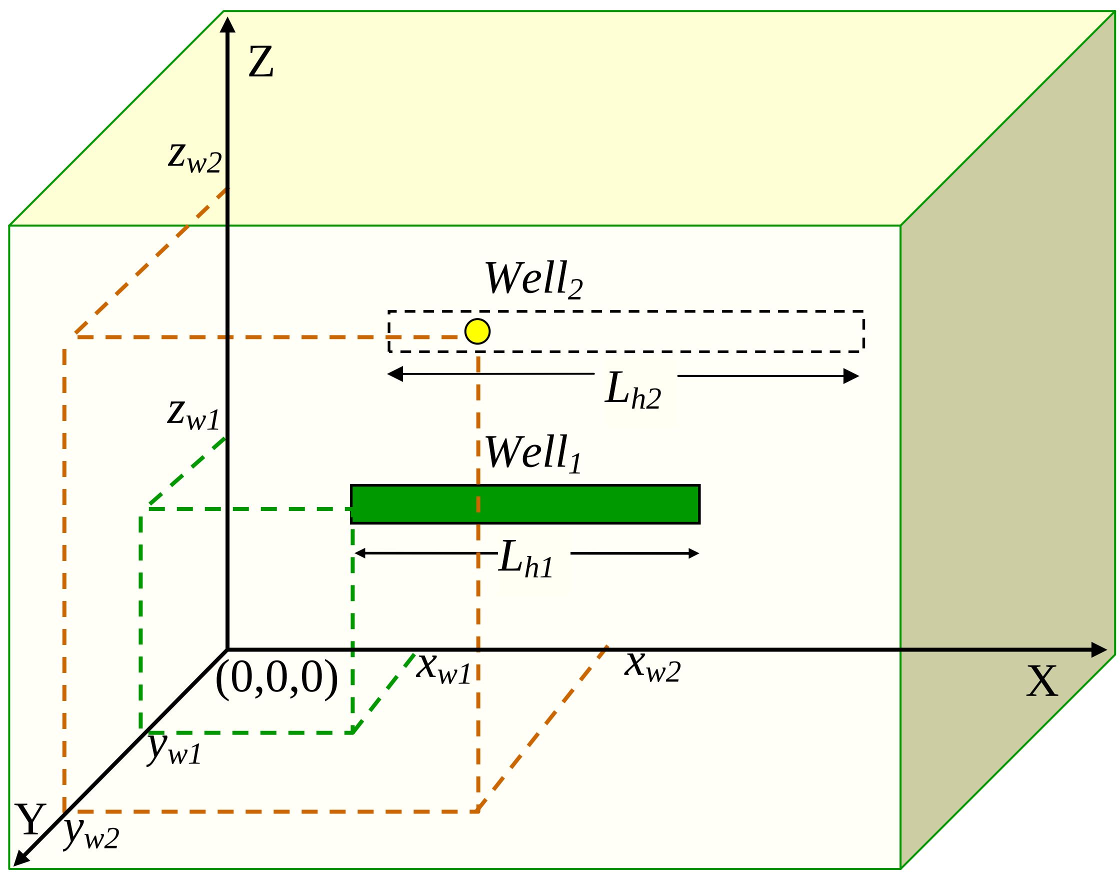

Figure-3.1 Well Configuration in 3-Dimension

In Figure 3-1 the two wells are parallel and aligned with the permeability in the x-direction but are not equal in length. The lengths of the active and observation wells are respectively L_1 and L_2. The heels of both wells are assumed to be their producing ends though the production at the heel of the observation well is zero. The coordinates of these heels are designated as x_{wi}, y_{wi}, z_{wi} where $i = 1$ or 2 refers to Well 1 or 2 respectively. Fluid can enter and leave both wells along their entire length. At any point x, the flow rate in well i, q_{hci}, is given by the following relationship [10, 11]:

$$q_{hci}\left(x,t\right) = \int_x^{L_{hi}+x_{wi}} q_{hi}\left(x',t\right) dx', \tag{3.1}$$

where q_{hi} denotes the flux (rate per unit length) entering or leaving well i at point x and at time t.

3.2 MATHEMATICAL MODEL

This model is a modification of the semi-analytical model previously developed by Al-Khamis et al. [17, 18]. This modification is done by redefining the dimensionless variables to take care of unequal lengths of the active and observation wells. In doing this, we introduce the ratio L_2 to L_1, denoted by R_D. We also choose the half length, l_1, of the active well as the characteristic length in defining the dimensionless variables. The choice of l_1 as the characteristic length renders all derivations in the previous model valid for the active well. We however, need to make adjustment to the previously derived model [17, 18, 19] to take care of the length of the observation well which is considered here to be unequal to that of the active well. We start by redefining the dimensionless pressure and time as follows:

$$P_D = \frac{kh}{141.2 q \mu B} \Delta P, \tag{3.2}$$

and

$$t_D = \frac{2.637 \times 10^{-4} kt}{\mu \phi c_t l_1^2} \tag{3.3}$$

for t in hours. The horizontal wells' half lengths are defined as.

$$l_1 = \frac{L_1}{2}, \tag{3.4}$$

and

$$l_2 = \frac{L_2}{2}. \tag{3.5}$$

Other dimensionless variables are defined as follow.

$$x_D = \frac{x}{l_1} \sqrt{\frac{k}{k_x}}, \tag{3.6}$$

$$x_{wD} = \frac{x_w}{l_1} \sqrt{\frac{k}{k_x}}, \tag{3.7}$$

$$y_D = \frac{y}{l_1}\sqrt{\frac{k}{k_x}}, \tag{3.8}$$

$$y_{wD} = \frac{y_w}{l_1}\sqrt{\frac{k}{k_x}}, \tag{3.9}$$

$$z_D = \frac{z}{h}, \tag{3.10}$$

and

$$z_{wD} = \frac{z_w}{h}. \tag{3.11}$$

Dimensionless well lengths are defined as

$$L_{D1} = \frac{l_1}{h}\sqrt{\frac{k_z}{k}}, \tag{3.12}$$

and

$$L_{D2} = \frac{l_2}{h}\sqrt{\frac{k_z}{k}}. \tag{3.13}$$

We also introduce a new ratio between the two wells as

$$R_D = \frac{l_2}{l_1}. \tag{3.14}$$

The dimensionless formation thickness and dimensionless wellbore radius are given by

$$h_D = \frac{h}{l_1}\sqrt{\frac{k}{k_z}} \tag{3.15}$$

and

$$r_{wD} = \frac{r_{we}}{h}, \text{ respectively.} \tag{3.16}$$

In equation (3.16) r_{we} is the equivalent wellbore radius for an anisotropic reservoir given by

$$r_{we} = \frac{r_w}{2}\left[\left(\frac{\sqrt{k_x k_y}}{k_z}\right)^{0.25} + \left(\frac{k_z}{\sqrt{k_x k_y}}\right)^{0.25}\right]. \tag{3.17}$$

The dimensionless fluxes in both wells are defined as

$$q_{hD1} = \frac{2q_{h1}l_1}{qB}\sqrt{\frac{k_x}{k}}, \tag{3.18}$$

and

$$q_{hD2} = \frac{2q_{h2}l_2}{qB}\sqrt{\frac{k_x}{k}}, \text{ respectively.} \tag{3.19}$$

As previously defined [10, 11, 17, 18, 19], we use the Reynolds number

$$N_{\mathrm{Re}c}(x) = 6.166*10^{-2}\,\frac{\rho q_{hc}(x)}{\mu r_w} \tag{3.20}$$

and dimensionless wellbore conductivities

$$C_{hD1} = 7.395*10^{13}\,\frac{r_{w1}^4}{khL_1} \tag{3.21}$$

and

$$C_{hD2} = 7.395*10^{13}\,\frac{r_{w2}^4}{khL_2}. \tag{3.22}$$

The roughness factor is given [19] as

$$\varepsilon_D = \frac{\varepsilon}{2r_w}. \tag{3.23}$$

3.3 Semi-Analytical Model

This is based on the superposition of two finite-conductivity horizontal well solutions for the conceptual model described above. The coupled wellbore-reservoir model for the active well remains as developed in the literature [10, 11] while that of the horizontal observation well is adjusted to account for unequal length. The reservoir flow equation for the active well is given by:

$$\Delta P_1 = \frac{1}{4\pi\phi c_t h}\int_0^t\int_0^{L_1}\frac{q_{h1}\left(x',\tau\right)}{\left(t-\tau\right)\sqrt{\eta_x\eta_y}}\exp\left[\frac{-\left(x-x_w-x'\right)^2}{4\eta_x\left(t-\tau\right)}\right]dx'\exp\left[\frac{-\left(y-y_w\right)^2}{4\eta_y\left(t-\tau\right)}\right]$$
$$\left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2\eta_z\left(t-\tau\right)}{h^2}\right]\cos n\pi\frac{z_w}{h}\cos n\pi\frac{z}{h}\right\}d\tau. \quad (3.24)$$

We may express equation (3.24) in dimensionless form as:

$$P_{D1} = \frac{1}{4}\sqrt{\frac{k^2}{k_x k_y}}\int_0^{t_D}\int_0^{2L_{D1}h_D\sqrt{\frac{k}{k_x}}} q_{hD1}\left(\alpha,\tau_D\right)\exp\left[\frac{-\left(x_D-x_{wD}-\alpha\right)^2}{4\left(t_D-\tau_D\right)}\right]d\alpha$$
$$\exp\left[\frac{-\left(y_D-y_{wD}\right)^2}{4\left(t_D-\tau_D\right)}\right]\left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2\left(t_D-\tau_D\right)}{{h_D}^2}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}d\tau_D. \quad (3.25)$$

3.4 Coupled Wellbore-Reservoir Model for Active Well

As previously derived in the literature [10, 11], the steady-state momentum equation for single-phase isothermal flow in a horizontal well is given by

$$\frac{dP_{h1}}{dx} = Ef_1 {q_{hc1}}^2. \tag{3.26}$$

P_{h1} represents the pressure at some point in the horizontal Well 1, f is the fanning friction factor and

$$E = 9.117 * 10^{-13} \frac{\rho}{\pi^2 {r_w}^5}. \tag{3.27}$$

Differentiating equation (3.26) with respect to x, we obtain

$$\frac{d^2 P_{h1}}{dx^2} = E\left(q_{hc1}^2 \frac{df_1}{dx} - 2 f_1 q_{hc1} q_{h1} \right). \tag{3.28}$$

q_{hc1} is the flow rate in Well 1 at some point x and q_{h1} is the flux entering Well 1 at some point x. The flow rate and the flux are related by

$$\left(\frac{\partial q_{hc1}}{\partial x} \right)_{x,t} = -q_{h1}\left(x,t \right). \tag{3.29}$$

We now integrate equation (3.28) and use the following boundary condition

$$\left(\frac{dP_{hc}}{dx}\right)_{x=x_{w1}} = Ef_{t1}q^2B^2 \tag{3.30}$$

to obtain

$$\frac{dP_{h1}}{dx} = Ef_{t1}q^2B^2 + E\int_{x_{w1}}^{x}\left(q_{hc1}^2\frac{df_1}{dx'} - 2f_1q_{hc1}q_{h1}\right)dx'. \tag{3.31}$$

In equation (3.31) q is the production rate of Well 1, as measured at the surface, and f_{t1} is the friction factor at the heel of Well 1. Integrating equation (3.28) one more time yields

$$P_{h1}(x,t) - P_{wf1}(t) = Ef_{t1}q^2B^2(x - x_{w1}) + E\int_{x_{w1}}^{x}\int_{x_{w1}}^{x'}\left(q_{hc1}^2\frac{df_1}{dx''} - 2f_1q_{hc1}q_{h1}\right)dx''dx'. \tag{3.32}$$

Equation (3.32) represents the pressure drop in the wellbore at any particular time t. Equation (3.32) may be expressed in dimensionless form as as:

$$P_{wD1}(t_D) - P_{hD1}(x_D,t_D) = \frac{\pi N_{\mathrm{Re}t1}f_{t1}}{16C_{hD1}}\left[\frac{2(x_D - x_{wD1})\int_{x_{wD1}}^{x_D}\int_{x_{wD1}}^{x'_D}\left(\frac{D_1q_{hD1}}{N_{\mathrm{Re}t1}f_{t1}}\right)dx''_Ddx'_D}{L_{D1}h_D\sqrt{\frac{k}{k_x}}}\right]. \tag{3.33}$$

In equation (3.33) we have made use of the definitions below

$$\frac{N_{\mathrm{Re}1}}{N_{\mathrm{Re}t1} f_1}\left(\frac{df_1}{dx_D}\right) - q_{hD1} = -\frac{q_{hD1}}{2}\left(2 + \frac{N_{\mathrm{Re}1}}{f_1}\frac{df_1}{dN_{\mathrm{Re}1}}\right), \tag{3.34}$$

where $N_{\mathrm{Re}t1}$, the Reynolds number at the heel of the well ($x = 0$), is known because the flow rate $\left[q_{hc}\left(x = 0\right) = q\right]$ at that point is known. However, the flux varies along the horizontal section so that $N_{\mathrm{Re}}\left(x\right)$ will be a correlating group for the friction factor, f, and will determine the flow regime. D_1 is defined as:

$$D_1 = N_{\mathrm{Re}1}^2 \frac{df_1}{dN_{\mathrm{Re}1}} + 2 N_{\mathrm{Re}1} f_1. \tag{3.35}$$

P_{wD1} and P_{hD1} are the dimensionless pressure measured at the heel of the active well $\left(x_D = 0\right)$ and at some point x_D respectively.

Using the principle of superposition we may obtain the dimensionless pressure at some point in the reservoir (in this case, any point in the active well) by summing the pressure drops caused by the two horizontal wells (active and observation) as given by equation (3.36).

$$\begin{aligned} P_{hD1}\left(x_D, t_D\right) = {} & P_{D1}\left(x_D, y_{wD1}, z_{wD1} + r_{wD1}; x_{wD1}, y_{wD1}, z_{wD1}, t_D\right) + q_{hD1}\left(x_D, t_D\right) S_1\left(x_D\right) \\ & + P_{D2}\left(x_D, y_{wD1}, z_{wD1} + r_{wD1}; x_{wD2}, y_{wD2}, z_{wD2}, t_D\right) \end{aligned} \tag{3.36}$$

$S_1\left(x_D\right)$ is the mechanical skin around Well 1 at point x_D. The dimensionless pressure drop due to this skin has been given in the literature [26] as

$$P_{Ds} = q_{hD} L_{D1} h_D S_{hm} \sqrt{\frac{k}{k_x}}, \tag{3.37}$$

where the horizontal well skin factor is defined by Ozkan and Raghavan [20] as

$$S_{hm} = \frac{P_h\left(r_w, x, t\right) - P_s\left(r_w + \Delta r_{ws}, x, t\right)}{\frac{Lk_{\tilde{r}}}{kh}\left(\tilde{r}\frac{\partial P}{\partial \tilde{r}}\right)_{\tilde{r}=r_w, x}} = \frac{\frac{kh}{141.2 qB\mu}\Delta P_s}{\tilde{q}_{hD}}. \tag{3.38}$$

By continuity of flux along the well, the q_{hD1} terms in equations (3.33) and (3.36) are the same. Equations (3.33) and (3.36) therefore represent the coupled wellbore-reservoir flow model for the active well.

3.5 Reservoir Model for Observation Well

The reservoir fluid flow equation for Well 2 is given by the following.

$$\begin{aligned}\Delta P_2 = {} & \frac{1}{4\pi\phi c_t h}\int_0^t\int_0^{L_2}\frac{q_{h2}\left(x',\tau\right)}{\left(t-\tau\right)\sqrt{\eta_x\eta_y}}\exp\left[\frac{-\left(x-x_w-x'\right)^2}{4\eta_x\left(t-\tau\right)}\right]dx'\exp\left[\frac{-\left(y-y_w\right)^2}{4\eta_y\left(t-\tau\right)}\right] \\ & \left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2\eta_z\left(t-\tau\right)}{h^2}\right]\cos n\pi\frac{z_w}{h}\cos n\pi\frac{z}{h}\right\}d\tau.\end{aligned} \tag{3.39}$$

Using the previously stated definitions of the dimensionless variables leads to the following dimensionless reservoir fluid flow equation for the observation well.

$$
\begin{aligned}
P_{D2} = {} & \frac{1}{4R_D}\sqrt{\frac{k^2}{k_x k_y}}\int_0^{t_D}\int_0^{2R_D\sqrt{\frac{k}{k_x}}} q_{hD2}\left(\alpha,\tau_D\right)\exp\left[\frac{-\left(x_D - x_{wD} - \alpha\right)^2}{4\left(t_D - \tau_D\right)}\right]d\alpha \\
& \exp\left[\frac{-\left(y_D - y_{wD}\right)^2}{4\left(t_D - \tau_D\right)}\right] * \left\{1 + 2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2\left(t_D - \tau_D\right)}{{h_D}^2}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}d\tau_D.
\end{aligned}
\tag{3.40}
$$

We note that the main constraint in our definition of dimensionless variables is our choice of t_D which is defined using the active well's half length. This choice has led to the modification of the observation well reservoir equation as shown above. However, because we assume a steady-state flow in both horizontal wells, the wellbore equations in both wells are not affected by the choice of t_D. The derivations are given as follows

$$
\eta_x\left(t-\tau\right) = \frac{k_x\left(t-\tau\right)}{\mu\phi c_t} = \left(t_D - \tau_D\right)\frac{k_x}{k}{l_1}^2. \tag{3.41}
$$

The inverse of $\eta_x\left(t-\tau\right)$ is given by

$$
\frac{1}{\eta_x\left(t-\tau\right)} = \frac{1}{l_1^2\left(t_D - \tau_D\right)}\frac{k}{k_x}. \tag{3.42}
$$

A similar expression to equation (3.42) for diffusity in the y-direction is given by

$$\frac{1}{\eta_y(t-\tau)} = \frac{1}{l_1^2(t_D-\tau_D)}\frac{k}{k_y}, \tag{3.43}$$

and for the z -direction we have

$$\eta_z(t-\tau) = l_1^2(t_D-\tau_D)\frac{k_z}{k}. \tag{3.44}$$

The dimensionless variable α in equation (3.40) is given by

$$\alpha = \frac{x'}{l_1}\sqrt{\frac{k}{k_x}}. \tag{3.45}$$

We differentiate equation (3.45) with respect to x' to obtain

$$\frac{d\alpha}{dx'} = \frac{1}{l_1}\sqrt{\frac{k}{k_x}}. \tag{3.46}$$

Rearranging equation (3.46) leads to

$$dx' = l_1\sqrt{\frac{k_x}{k}}d\alpha. \tag{3.47}$$

The inverse of $\sqrt{\eta_x\eta_y}$ may be expressed as

$$\frac{1}{\sqrt{\eta_x \eta_y}} = \frac{1}{\sqrt{\frac{k_x}{\mu\phi c_t} \cdot \frac{k_y}{\mu\phi c_t}}} = \frac{\mu\phi c_t}{\sqrt{k_x k_y}}. \tag{3.48}$$

In equation (3.40) τ_D is given by

$$\tau_D = \frac{k\tau}{\mu\phi c_t l_1^{\,2}}. \tag{3.49}$$

Differentiating τ_D with respect to τ gives

$$\frac{d\tau_D}{d\tau} = \frac{k}{\mu\phi c_t l_1^{\,2}}, \tag{3.50}$$

and on rearranging we have

$$d\tau = \frac{\mu\phi c_t l_1^{\,2}}{k} d\tau_D. \tag{3.51}$$

We transform the limit of integration in equation (3.39) as follows

$$at \quad x' = 0, \quad \alpha = 0, \tag{3.52}$$

$$at \quad x' = L_2, \quad \alpha = \frac{L_2}{l_1}\sqrt{\frac{k}{k_x}} = 2R_D\sqrt{\frac{k}{k_x}} = 2L_{D2}h_D\sqrt{\frac{k}{k_x}}, \tag{3.53}$$

$$at \quad \tau = 0, \quad \tau_D = 0, \tag{3.54}$$

and

$$at \quad \tau = t, \quad \tau_D = t_D. \tag{3.55}$$

Substituting equations (3.42), (3.43), and (3.44) into equation (3.39) yields

$$\Delta P_2 = \frac{1}{4\pi\phi c_t h}\int_0^{t_D}\int_0^{2L_{D2}h_D\sqrt{\frac{k}{k_x}}} \frac{q_{h2}(\alpha,\tau)}{(t-\tau)}\frac{\mu\phi c_t}{\sqrt{k_x k_y}}$$
$$\exp\left[\frac{-1}{4(t_D-\tau_D)}\left(\frac{x-x_w-\alpha}{l_1}\sqrt{\frac{k}{k_x}}\right)^2\right] l_1\sqrt{\frac{k_x}{k}}d\alpha.\exp\left[\frac{-1}{4(t_D-\tau_D)}\left(\frac{y-y_w}{l_1}\sqrt{\frac{k}{k_y}}\right)^2\right]$$
$$\left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2}{h^2}l_1^{\,2}(t_D-\tau_D)\frac{k_z}{k}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}d\tau. \tag{3.56}$$

Simplifying equation (3.56) and further substituting equation (3.48) into it leads to

$$\Delta P_2 = \frac{1}{4\pi\phi c_t h}\frac{k}{\sqrt{k_x k_y}}\int_0^{t_D}\int_0^{2L_{D2}h_D\sqrt{\frac{k}{k_x}}}\frac{q_{h2}(\alpha,\tau)}{l_1^{\,2}\dfrac{k(t-\tau)}{\mu\phi c_t l_1^{\,2}}}\exp\left[\frac{-(x_D-x_{wD}-\alpha)^2}{4(t_D-\tau_D)}\right]l_1\sqrt{\frac{k_x}{k}}d\alpha$$
$$\exp\left[\frac{-(y_D-y_{wD})^2}{4(t_D-\tau_D)}\right]\left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2(t_D-\tau_D)}{\left(\dfrac{h}{l_1}\sqrt{\dfrac{k}{k_z}}\right)^2}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}d\tau. \tag{3.57}$$

We simplify further to obtain

$$\Delta P_2 = \frac{1}{4\pi\phi c_t h l_1^{\,2}}\sqrt{\frac{k^2}{k_x k_y}}\int_0^{t_D}\int_0^{2L_{D2}h_D\sqrt{\frac{k}{k_x}}}\frac{q_{h2}(\alpha,\tau)}{(t_D-\tau_D)}\exp\left[\frac{-(x_D-x_{wD}-\alpha)^2}{4(t_D-\tau_D)}\right]l_1\sqrt{\frac{k_x}{k}}d\alpha$$
$$\exp\left[\frac{-(y_D-y_{wD})^2}{4(t_D-\tau_D)}\right]\left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2(t_D-\tau_D)}{h_D^{\,2}}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}\frac{\mu\phi c_t l_1^{\,2}}{k}d\tau_D. \tag{3.58}$$

Further simplification and regrouping leads to

$$\Delta P_2 = \frac{\mu l_1}{4\pi kh}\sqrt{\frac{k^2}{k_x k_y}}\int_0^{t_D}\int_0^{2L_{D2}h_D\sqrt{\frac{k}{k_x}}}\frac{2l_2 q_{h2}(\alpha,\tau)}{qB}\sqrt{\frac{k_x}{k}}\left(\frac{qB}{2l_2}\right)\exp\left[\frac{-(x_D-x_{wD}-\alpha)^2}{4(t_D-\tau_D)}\right]d\alpha$$
$$\exp\left[\frac{-(y_D-y_{wD})^2}{4(t_D-\tau_D)}\right]\left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2(t_D-\tau_D)}{h_D^{\,2}}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}\frac{d\tau_D}{(t_D-\tau_D)}. \tag{3.59}$$

We substitute equation (3.19) into equation (3.59) to obtain

$$\Delta P_2 = \frac{q\mu B l_1}{8\pi k h l_2}\sqrt{\frac{k^2}{k_x k_y}}\int_0^{t_D}\int_0^{2L_{D2}h_D\sqrt{\frac{k}{k_x}}} q_{hD2}\left(\alpha,\tau_D\right)\exp\left[\frac{-\left(x_D - x_{wD} - \alpha\right)^2}{4\left(t_D-\tau_D\right)}\right]d\alpha$$
$$\exp\left[\frac{-\left(y_D - y_{wD}\right)^2}{4\left(t_D-\tau_D\right)}\right]\left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2\left(t_D-\tau_D\right)}{{h_D}^2}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}\frac{d\tau_D}{\left(t_D-\tau_D\right)}. \tag{3.60}$$

Substituting equation (3.14) into (3.60) and further simplifying gives the reservoir model for the observation well as

$$P_{D2} = \frac{1}{4R_D}\sqrt{\frac{k^2}{k_x k_y}}\int_0^{t_D}\int_0^{2L_{D2}h_D\sqrt{\frac{k}{k_x}}} q_{hD2}\left(\alpha,\tau_D\right)\exp\left[\frac{-\left(x_D - x_{wD} - \alpha\right)^2}{4\left(t_D-\tau_D\right)}\right]d\alpha$$
$$\exp\left[\frac{-\left(y_D - y_{wD}\right)^2}{4\left(t_D-\tau_D\right)}\right]\left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2\left(t_D-\tau_D\right)}{{h_D}^2}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}\frac{d\tau_D}{\left(t_D-\tau_D\right)}. \tag{3.61}$$

If we divide the well into M segments of equal length, and assume the flow rate to be constant in each segment, the reservoir model for Well 2 may be evaluated for each segment. We may thus evaluate the integral in equation (3.39) as follows.

$$\int_0^{L_2}\exp\left[\frac{-\left(x-x_w-x'\right)^2}{4\eta_x\left(t\right)}\right]dx' = \int_{-l2}^{l_2}\exp\left[\frac{-\left(x-x_w-x'\right)^2}{4\eta_x\left(t\right)}\right]dx' \tag{3.62}$$

where $l_2 = \frac{L_2}{2}$.

Using co-ordinate transformation we let

$$u^2 = \frac{\left(x - x_w - x'\right)^2}{4\eta_x t}, \tag{3.63}$$

so that

$$u = \frac{\left(x - x_w - x'\right)}{2\sqrt{\eta_x t}}, \tag{3.64}$$

and

$$\frac{du}{dx'} = \frac{-1}{2\sqrt{\eta_x t}}. \tag{3.65}$$

Rearranging equation (3.65) gives

$$dx' = -2\sqrt{\eta_x t}\,du. \tag{3.66}$$

We also transform the limits of integration as

$$\text{at } x' = -l_2, \quad u = \frac{x - x_w + l_2}{2\sqrt{\eta_x t}}, \tag{3.67}$$

and

at $x' = +l_2, \quad u = \dfrac{x - x_w - l_2}{2\sqrt{\eta_x t}}.$ (3.68)

We substitute equations (3.63) to (3.68) into equation (3.62) to obtain

$$\int_{-l_2}^{l_2} \exp\left[\frac{-\left(x - x_w - x'\right)^2}{4\eta_x t}\right] dx' = \int_{\frac{x - x_w + l_2}{2\sqrt{\eta_x t}}}^{\frac{x - x_w - l_2}{2\sqrt{\eta_x t}}} -2\sqrt{\eta_x t}\, e^{-u^2}\, du \tag{3.69}$$

$$= \sqrt{\eta_x \pi t} * \frac{2}{\sqrt{\pi}} \int_{\frac{x - x_w - l_2}{2\sqrt{\eta_x t}}}^{\frac{x - x_w + l_2}{2\sqrt{\eta_x t}}} e^{-u^2}\, du \tag{3.70}$$

$$= \sqrt{\eta_x \pi t}\left[erf\left(\frac{x - x_w + l_2}{2\sqrt{\eta_x t}}\right) - erf\left(\frac{x - x_w - l_2}{2\sqrt{\eta_x t}}\right)\right]. \tag{3.71}$$

Substituting equation (3.72) into equation (3.39) gives

$$\begin{aligned}\Delta P_2 = \frac{q_{h2}}{4\pi\phi c_t h} \int_0^{\tau} \frac{\sqrt{\pi \eta_x t}}{t\sqrt{\eta_x \eta_y}} \left[erf\left(\frac{x - x_w + l_2}{2\sqrt{\eta_x t}}\right) - erf\left(\frac{x - x_w - l_2}{2\sqrt{\eta_x t}}\right)\right] \\ \exp\left[\frac{-\left(y - y_w\right)^2}{4\eta_y t}\right] \left\{1 + 2\sum_{n=1}^{\infty} \exp\left[\frac{-n^2\pi^2\eta_z t}{h^2}\right] \cos n\pi \frac{z_w}{h} \cos n\pi \frac{z}{h}\right\} dt.\end{aligned} \tag{3.72}$$

Introducing the previously defined dimensionless variables into equation (3.72) leads to

$$\Delta P_2 = \frac{q_{h2}\sqrt{\pi}}{4\pi\phi c_t h\sqrt{\frac{k_y}{\mu\phi c_t}}}\int_0^{\tau_D}\frac{1}{\sqrt{t_D}\,l_1\sqrt{\frac{\mu\phi c_t}{k}}}\exp\left[\frac{-\left(y_D - y_{wD}\right)^2}{4t_D}\right]$$

$$\left[erf\left(\frac{x_D - x_{wD} + L_{D2}h_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right) - erf\left(\frac{x_D - x_{wD} - L_{D2}h_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right)\right] \quad (3.73)$$

$$\left\{1 + 2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2 t_D}{h_D^{\ 2}}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}\left(\frac{\mu\phi c_t l_1^{\ 2}}{k}\right)dt_D.$$

Simplifying further gives

$$\Delta P_2 = \frac{q_{h2}\mu l_1}{4\sqrt{\pi}kh}\sqrt{\frac{k}{k_y}}\int_0^{\tau_D}\left[erf\left(\frac{x_D - x_{wD} + R_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right) - erf\left(\frac{x_D - x_{wD} - R_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right)\right] \quad (3.74)$$

$$\exp\left[\frac{-\left(y_D - y_{wD}\right)^2}{4t_D}\right]\left\{1 + 2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2 t_D}{h_D^{\ 2}}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}\frac{dt_D}{\sqrt{t_D}}.$$

Introducing q_{hD2} into equation (3.74) leads to:

$$\Delta P_2 = \frac{q_{hD2}\mu l_1 qB}{8l_2\sqrt{\pi}kh}\sqrt{\frac{k}{k_x}}\sqrt{\frac{k}{k_y}}\int_0^{\tau_D}\left[erf\left(\frac{x_D - x_{wD} + R_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right) - erf\left(\frac{x_D - x_{wD} - R_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right)\right]$$

$$\exp\left[\frac{-\left(y_D - y_{wD}\right)^2}{4t_D}\right]\left\{1 + 2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2 t_D}{h_D^{\ 2}}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}\frac{dt_D}{\sqrt{t_D}}.$$

(3.75)

Substituting equation (3.75) into equation (3.2) gives

$$P_{D2} = \frac{2\pi kh}{q\mu B} \cdot \frac{q\mu Bq_{hD2}}{8R_D\sqrt{\pi}kh}\sqrt{\frac{k^2}{k_xk_y}}\int_0^{\tau_D}\left[erf\left(\frac{x_D - x_{wD} + R_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right) - erf\left(\frac{x_D - x_{wD} - R_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right)\right]$$
$$\exp\left[\frac{-\left(y_D - y_{wD}\right)^2}{4t_D}\right]\left\{1 + 2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2t_D}{h_D{}^2}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}\frac{dt_D}{\sqrt{t_D}}.$$

(3.76)

Simplifying further gives

$$P_{D2} = \frac{q_{hD2}\sqrt{\pi}}{4R_D}\sqrt{\frac{k^2}{k_xk_y}}\int_0^{\tau_D}\left[erf\left(\frac{x_D - x_{wD} + R_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right) - erf\left(\frac{x_D - x_{wD} - R_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right)\right]$$
$$\exp\left[\frac{-\left(y_D - y_{wD}\right)^2}{4t_D}\right]\left\{1 + 2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2t_D}{h_D{}^2}\right]\cos n\pi z_{wD}\cos n\pi z_D\right\}\frac{dt_D}{\sqrt{t_D}}. \quad (3.77)$$

Equation (3.77) represents the reservoir model for the observation well. We need to couple this model with the wellbore model for observation well.

3.6 Coupled Wellbore-Reservoir Model for the Observation Well

Following the approach developed by Al-Khamis et al. [17, 18, 19] we can derive the wellbore equation for the observation well as presented below. Because there is no flow at the heel of the observation well we may write the boundary condition at the heel as:

$$\left(\frac{dP_{h2}}{dx}\right)_{x=x_{w2}} = 0. \tag{3.78}$$

Using the boundary condition in equation (3.78) the wellbore equation for the observation well is obtained as:

$$P_{h2}(x,t) - P_{wf\,2}(t) = E\int_{x_{w2}}^{x}\int_{x_{w2}}^{x'}\left(q_{hc2}^{2}\frac{df_2}{dx''} - 2f_2 q_{hc2} q_{h2}\right)dx''dx'. \tag{3.79}$$

In dimensionless form equation (3.79) becomes

$$P_{wD2}(t_D) - P_{hD2}(x_D, t_D) = \frac{-\pi}{16C_{hD2}}\left[\frac{\int_{x_{wD1}}^{x_D}\int_{x_{wD1}}^{x'_D} D_2 q_{hD2}\, dx''_D dx'_D}{R_D\sqrt{\dfrac{k}{k_x}}}\right], \tag{3.80}$$

where D_2 is given by

$$D_2 = N_{\mathrm{Re}2}^{2}\frac{df_2}{dN_{\mathrm{Re}2}} + 2N_{\mathrm{Re}2} f_2. \tag{3.81}$$

The dimensionless pressure at any point in Well 2 then becomes:

$$P_{hD2}(x_D, t_D) = P_{D1}(x_D, y_{wD2}, z_{wD2} + r_{wD2}; x_{wD1}, y_{wD1}, z_{wD1}, t_D) + q_{hD2}(x_D, t_D) S_2(x_D)$$

$$+P_{D2}\left(x_D, y_{wD2}, z_{wD2}+r_{wD2}; x_{wD2}, y_{wD2}, z_{wD2}, t_D\right). \tag{3.82}$$

In equation (3.82), $S_2\left(x_D\right)$ is the skin distribution along Well 2 and is given by

$$S_2\left(x_D\right)=\frac{\frac{kh}{141.2qB\mu}\Delta P_{2s}\left(x,t\right)}{\tilde{q}_{hD2}\left(x_D,t_D\right)}. \tag{3.83}$$

Equations (3.33) and (3.80) are the coupled flow equations for finite-conductivity active and observation wells respectively. It is not expected that there would be considerable frictional pressure drop in the observation well because the flow rate along the horizontal section of this well is small.

We shall now proceed to develop the algorithm needed to estimate the flux distribution in both wells. This is necessary to be able to compute the pressure distribution in the reservoir.

3.7 Flux Distribution Estimation

If we assume, for convenience, that the center of the coordinate axis in x-y plane is at the heel of well 1 so that $x_{w1}=y_{w1}=0$, then equation (3.33) simplifies to

$$P_{wD1}\left(t_D\right)-P_{hD1}\left(x_D,t_D\right)=\frac{\pi N_{\mathrm{Re}t1} f_{t1}}{16C_{hD1}}\left[\frac{2x_D\int_{x_{wD1}}^{x_D}\int_{x_{wD1}}^{x'_D}\left(\frac{D_1 q_{hD1}}{N_{\mathrm{Re}t1} f_{t1}}\right)dx''_D dx'_D}{L_{D1}h_D\sqrt{\frac{k}{k_x}}}\right]. \tag{3.84}$$

At this coordinate center $P_{hD1}\left(x_D,t_D\right)$ may be written as

$$P_{hD1}\left(x_D,t_D\right)=P_{D1}\left(x_D,0,z_{wD1}+r_{wD1};0,0,z_{wD1},t_D\right)+q_{hD1}\left(x_D,t_D\right)S_1\left(x_D\right)$$

$$+P_{D2}\left(x_D,0,z_{wD1}+r_{wD1};x_{wD2},y_{wD2},z_{wD2},t_D\right) \tag{3.85}$$

We shall now divide well 1 into M segments and denoting the center of each segment as x_{Dj}, we may write the double integral in equation (3.84) as

$$\int_{x_{wD1}}^{x_D}\int_{x_{wD1}}^{x'_D} D_1 q_{hD1} dx''_D dx'_D=\sum_{i=1}^{j-1}\left(x_{Dj}-i\Delta x_{D1}+\frac{\Delta x_{D1}}{2}\right)\Delta x_{D1}D_{1i}q_{hD1i}+\frac{\Delta x_{D1}^2}{8}D_{1j}q_{hD1j} \tag{3.86}$$

In equation (3.86) D_{1i} and q_{hD1i} are evaluated at the centre of the i^{th} interval.

We shall divide both wells into M equal segments to obtain the dimensionless Green's function for Well 1 as

$$G_{D1i}\left(x_D,y_D,z_D;x_{wD1},y_{wD1},z_{wD1},t_D\right)=\frac{\sqrt{\pi}}{4}\sqrt{\frac{k^2}{k_xk_y}}\int_0^{\tau_D}\exp\left[\frac{-\left(y_D-y_{wD1}\right)^2}{4t_D}\right]$$
$$\left[erf\left(\frac{x_D-x_{wD1}-\frac{i-1}{M}L_{D1}h_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right)-erf\left(\frac{x_D-x_{wD1}-\frac{i}{M}L_{D1}h_D\sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}}\right)\right]$$
$$\left\{1+2\sum_{n=1}^{\infty}\exp\left[\frac{-n^2\pi^2t_D}{h_D{}^2}\right]\cos n\pi z_{wD1}\cos n\pi z_D\right\}\frac{dt_D}{\sqrt{t_D}}, \tag{3.87}$$

and for Well 2,

$$G_{D2i}\left(x_D, y_D, z_D; x_{wD2}, y_{wD2}, z_{wD2}, t_D\right) = \frac{\sqrt{\pi}}{4R_D}\sqrt{\frac{k^2}{k_x k_y}}\int_0^{\tau_D} \exp\left[\frac{-\left(y_D - y_{wD2}\right)^2}{4t_D}\right]$$
$$\left[erf\left(\frac{x_D - x_{wD2} - \frac{i-1}{M} R_D \sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}} \right) - erf\left(\frac{x_D - x_{wD2} - \frac{i}{M} R_D \sqrt{\frac{k}{k_x}}}{2\sqrt{t_D}} \right) \right] \tag{3.88}$$
$$\left\{1 + 2\sum_{n=1}^{\infty} \exp\left[\frac{-n^2\pi^2 t_D}{h_D{}^2}\right] \cos n\pi z_{wD2} \cos n\pi z_D \right\} \frac{dt_D}{\sqrt{t_D}}.$$

Using k to denote Well 1 or 2 we may now write P_{Dk} as follows

$$P_{Dk}\left(x_{Dj}, y_D, z_D; x_{wDk}, y_{wDk}, z_{wDk}, t_D\right) = \int_0^{t_D} \sum_{i=1}^{M} q_{hDki}\left(\tau_D\right)$$
$$G_{Dki}\left(x_{Dj}, y_D, z_D; x_{wDk}, y_{wDk}, z_{wDk}, t_D - \tau_D\right) d\tau_D. \tag{3.89}$$

Equation (3.89) may be discretized in time to obtain

$$P_{Dk}\left(x_{Dj}, y_D, z_D; x_{wDk}, y_{wDk}, z_{wDk}, t_D\right) = \sum_{l=1}^{N} \int_{t_{Dl-1}}^{t_{Dl}} \sum_{i=1}^{M} q_{hDki}\left(\tau_D\right)$$
$$G_{Dki}\left(x_{Dj}, y_D, z_D; x_{wDk}, y_{wDk}, z_{wDk}, t_D - \tau_D\right) d\tau_D. \tag{3.90}$$

We shall now assume q_{hDki} to be constant in each time interval $\left(t_{Dl}, t_{Dl-1}\right)$ so that we obtain

$$P_{Dk}\left(x_{Dj},y_D,z_D;x_{wDk},y_{wDk},z_{wDk},t_D\right)=\tilde{P}_{Dk}\left(x_{Dj},y_D,z_D;x_{wDk},y_{wDk},z_{wDk},t_D\right)$$
$$+\sum_{i=1}^{M}q_{hDki}\left(\tau_{DN}\right)\int_0^{t_{DN}-t_{DN-1}}G_{Dki}\left(x_{Dj},y_D,z_D;x_{wDk},y_{wDk},z_{wDk},t_D-\tau_D\right)d\tau_D. \tag{3.91}$$

In equation (3.91), we have used

$$\tilde{P}_{Dk}\left(x_{Dj},y_D,z_D;x_{wDk},y_{wDk},z_{wDk},t_D\right)=\sum_{l=1}^{N-1}\sum_{i=1}^{M}q_{hDki}\left(t_{Dl}\right)$$
$$\left[\int_0^{t_{DN}-t_{Dl-1}}G_{Dki}\left(x_{Dj},y_D,z_D;x_{wDk},y_{wDk},z_{wDk},\tau_D\right)d\tau_D-\int_0^{t_{DN}-t_{Dl}}G_{Dki}\left(x_{Dj},y_D,z_D;x_{wDk},y_{wDk},z_{wDk},\tau_D\right)d\tau_D\right]. \tag{3.92}$$

After substituting equations (3.85), (3.86), (3.91), and (3.92) into equations (3.84) we obtain

$$\begin{aligned}
&P_{wD1}\left(t_{DN}\right)-\tilde{P}_{D1}\left(x_{D1j},0,z_{wD1}+r_{wD1};0,0,z_{wD1},t_D\right)\\
&-\sum_{i=1}^{M}q_{hD1i}\left(t_{DN}\right)\int_0^{t_{DN}-t_{DN-1}}G_{D1i}(x_{D1j},0,z_D;0,0,z_{wDk},\tau_D)d\tau_D\\
&-q_{hD1j}S_{1j}-\tilde{P}_{D2}\left(x_{D1j},0,z_{wD1}+r_{wD1};x_{wD2j},y_{wD2j},z_{wD2},t_D\right)\\
&-\sum_{i=1}^{M}q_{hD2i}\left(t_{DN}\right)\int_0^{t_{DN}-t_{DN-}1}G_{D2i}\left(x_{D1j},0,z_{wD1}+r_{wD1};x_{wD2j},y_{wD2j},z_{wD2},\tau_D\right)d\tau_D=\\
&\frac{\pi N_{\mathrm{Ret}1}f_{t1}}{8C_{hD1}}\cdot\frac{1}{L_{D1}h_D\sqrt{\frac{k}{k_x}}}x_{D1j}\\
&-\frac{\pi}{16C_{hD1}}\cdot\frac{1}{L_{D1}h_D\sqrt{\frac{k}{k_x}}}\left[\sum_{i=1}^{j-i}\left(x_{Dij}-i\Delta x_{D1}+\frac{\Delta x_{D1}}{2}\right)\Delta x_{D1}D_{1i}q_{hD1i}+\frac{\left(\Delta x_{D1}\right)^2}{8}D_{1j}q_{hD1j}\right].
\end{aligned} \tag{3.93}$$

The reference point in the above is the heel of the active well. Thus we have

$$S_{1j} = S_1\left(x_{D1j}\right).$$

Similarly, we may write the above for Well 2 as

$$\begin{aligned}
&P_{wD2}\left(t_{DN}\right) - \tilde{P}_{D1}\left(x_{D2j}, 0, z_{wD2} + r_{wD2}; x_{wD1}, y_{wD1}, z_{wD1}, t_D\right) \\
&-\sum_{i=1}^{M} q_{hD1i}\left(t_{DN}\right) \int_0^{t_{DN}-t_{DN-1}} G_{D1i}(x_{D2j}, 0, z_{wD} + r_{wD2}; x_{wD1}, y_{wD1}, z_{wD1}, \tau_D) d\tau_D \\
&-\tilde{P}_{D2}\left(x_{D2j}, 0, z_{wD2} + r_{wD2}; 0, 0, z_{wD2}, t_D\right) \\
&-\sum_{i=1}^{M} q_{hD2i}\left(t_{DN}\right) \int_0^{t_{DN}-t_{DN-}1} G_{D2i}\left(x_{D2j}, 0, z_{wD2} + r_{wD2}; 0, 0, z_{wD2}, \tau_D\right) d\tau_D - q_{hD2j} S_{2j} = \\
&\frac{\pi}{16 C_{hD2}} . \frac{1}{R_D \sqrt{\frac{k}{k_x}}} \left[\sum_{i=1}^{j-i} \left(x_{D2j} - i\Delta x_{D2} + \frac{\Delta x_{D2}}{2} \right) \Delta x_{D2} D_{2i} q_{hD2i} + \frac{\left(\Delta x_{D2}\right)^2}{8} D_{2j} q_{hD2j} \right].
\end{aligned} \tag{3.94}$$

In this second case, the reference point is at the heel of Well 2.

To solve equations (3.93) and (3.94), we discretize both horizontal wells into M segments and evaluate equations (3.93) and (3.94) at the center of each segment $i\left(x_{Dki}\right)$. The discretized equations yield $2M$ in $2M+2$ unknowns. These unknowns are $P_{wD1}, P_{wD2}, q_{hD1i}$, and q_{hD2i} where $i = 1, M$. We need two additional expressions to solve the set of simultaneous equations arising from this algorithm. These are obtained by satisfying the condition that the sum of fluxes along the active well must be equal to the production rate and the sum of fluxes along the observation well must be zero. These conditions are expressed as:

$$\sum_{i=1}^{M} q_{hD1i} = \frac{M}{L_{D1} h_D \sqrt{\frac{k}{k_x}}}, \tag{3.95}$$

$$\sum_{i=1}^{M} q_{hD2i} = 0, \tag{3.96}$$

This system of non-linear simultaneous equations needs to be solved by an iterative procedure. For detailed solution algorithm and solution procedure readers are referred to references 18 and 19.

Chapter 4

MODEL VALIDATION

This chapter presents a validation of the modified model presented in this work and presents results derived from the investigation of various cases. Validation of results was carried out using two models. The two asymptotic cases used for model validation are:

1. Interference testing using two finite-conductivity horizontal wells of equal length [17, 18, 19].
2. Interference testing between a finite-conductivity horizontal well and a vertical observation well [10, 11].

4.1 Validation Using Two Horizontal Wells of Equal Length

Al-Khamis M. N., Ozkan, E. and Raghavan R. [17, 18, 19] presented a semi-analytic model that effectively takes care of interference testing between two finite-conductivity horizontal wells of equal length. The model presented here however is able to handle the same scenario when the two wells are of equal or unequal length. Since our work is an extension of Al-Khamis et al's model, it must give the same result as its precursor whenever we assume the two wells to be of equal length. The data in the Table 4.1 have been used to validate the new model.

Table 4.1- Data Used for Model Validation Using Two Horizontal Wells of Equal Length

Well parameters	Well 1	Well 2
$L,\ ft$	2000	2000
$r_w,\ ft$	0.165	0.165
$x_w,\ ft$	0.000	2800
$y_w,\ ft$	0.000	0.000
$z_w,\ ft$	50	50
N_{Re_t}	23889.9	-
Reservoir Parameters		
$h,\ ft$	100	
k_x, md	300	
k_y, md	300	
k_z, md	100	

We present a comparison between our model and Al-Khamis et al.'s model [17, 18] in Figure 4.1.

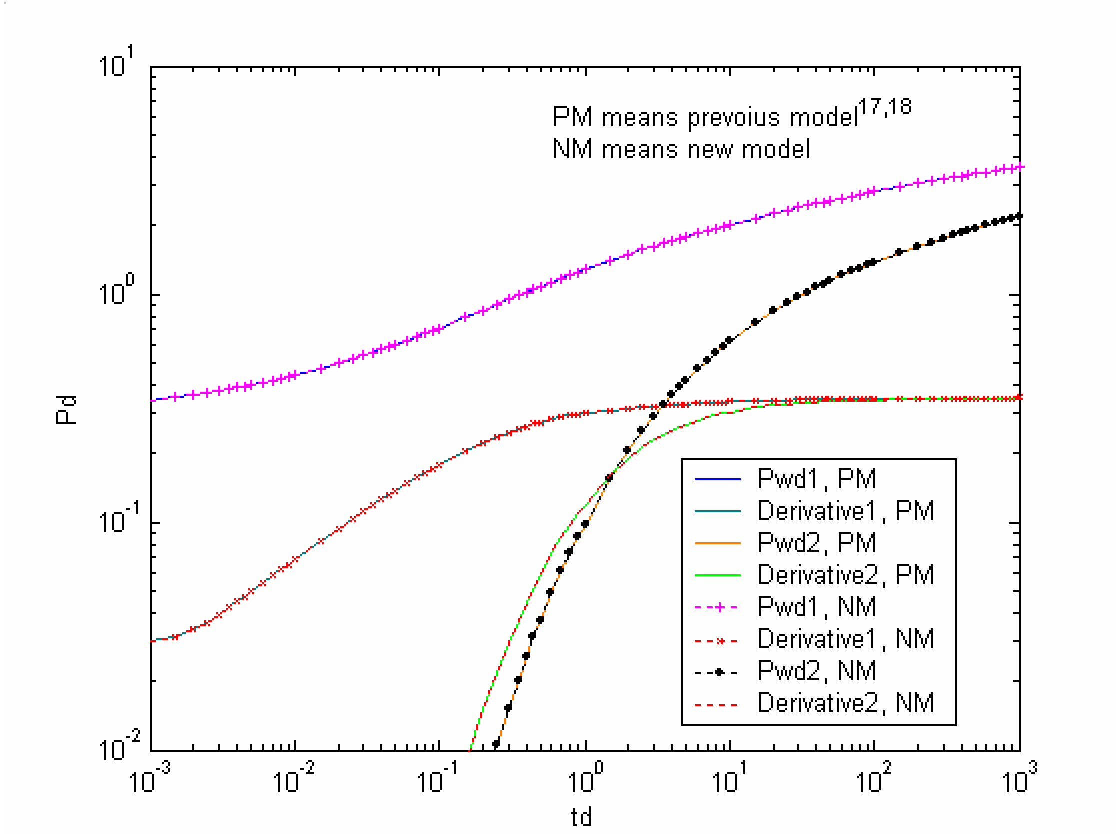

Figure 4.1- Validation of our model using Al-Khamis et al.'s model.

Figure 4.1 shows that the modified model is in excellent agreement with the original model.

4.2 Validation Using Horizontal-Vertical Interference Test Model

Here we compare our modified model with the model proposed by Ozkan et al. [10, 11]. The model considers interference test between a finite-conductivity horizontal well and a vertical observation well. Since our modified model has the flexibility of varying the observation well length while keeping the active well length constant, we are able to generate dimensionless pressure response when the horizontal observation well is shorten to its radius r_w. We use the same parameters used above except that the length of the observation well was assumed equal to its radius, i.e., $L_2 = r_{w2}$. Table 4.2 presents the data used for validating our model with Ozkan et al.

Table 4.2- Data Used for Model Validation Horizontal-Vertical Well Interference Test Model

Well parameters	Well 1	Well 2
$L,\ ft$	2000	0.165
$r_w,\ ft$	0.165	0.165
$x_w,\ ft$	0.000	2800
$y_w,\ ft$	0.000	0.000
$z_w,\ ft$	50	50
N_{Re_t}	23889.9	-
Reservoir Parameters		
$h,\ ft$	100	
k_x, md	300	
k_y, md	300	
k_z, md	100	

Our model is able to match the Ozkan et al.'s model with very good accuracy as can be seen from Figure 4.2.

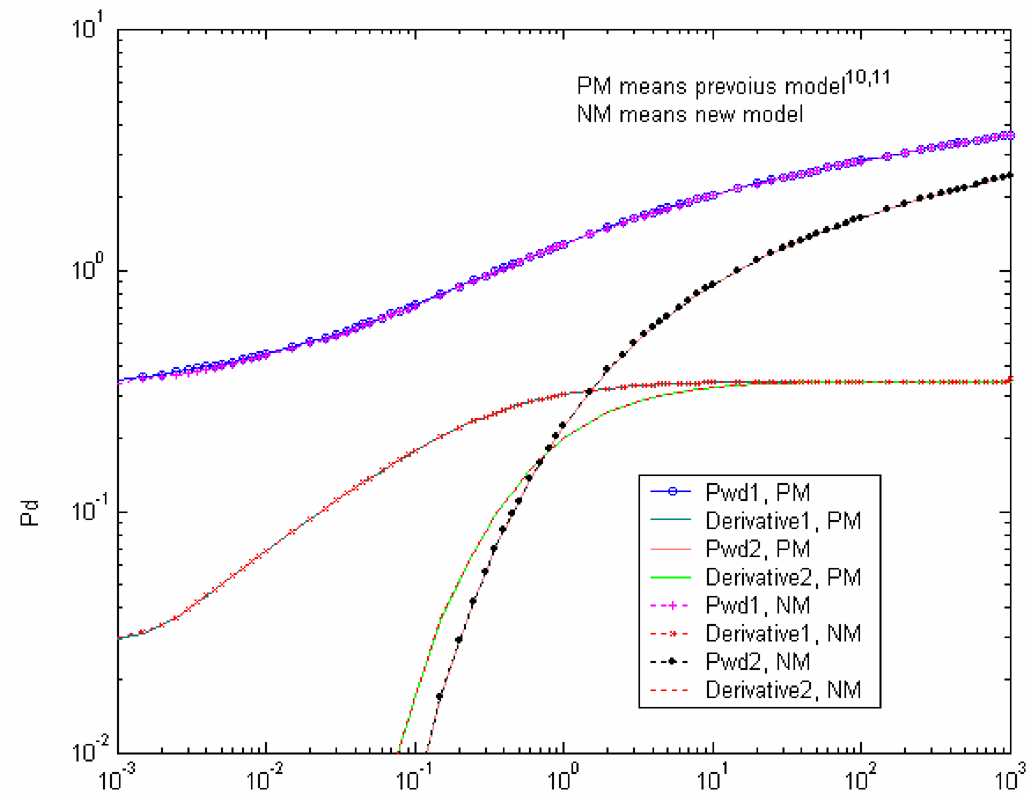

Figure 4.2- Validation of our model using Ozkan et al.'s model.

CHAPTER 5

RESULTS AND DISCUSSION

Here we present our study on the effect of changing well length on flux distribution, pressure response and appearance of flow regimes in both wells. The results presented show the effect of changing well lengths on the pressure response and flux distribution in the active and observation wells.

5.1 Flux Distribution

Flux distribution in active and observation horizontal wells have been studied by previous researchers for several cases. However, the effects of changing length on flux distribution in both wells have not been studied. Because the modified model can handle varying horizontal well length we are able to study this effect. We consider flux distribution in the observation well at late times only because it has been shown [19] that there is no flux distribution in the observation well at early times.

5.1.1 Effect of Changing Observation Well Length

In order to study the effect of observation well length on flux distribution in the active and observation wells, the active well length is held constant while the observation well length has been varied. Table 5.1 presents the data used to study the effect of observation well length on flux distribution.

Table 5.1- Data Used to Study Flux Distribution (Offset in x-direction)

Well parameters	Well 1	Well 2
$L,\ ft$	2000	0.165, 1000, 2000, 4000
$r_w,\ ft$	0.165	0.165
$x_w,\ ft$	0.000	2800
$y_w,\ ft$	0.000	0.000
$z_w,\ ft$	50	50
N_{Re_t}	23889.9	-
Reservoir Parameters		
$h,\ ft$	100	
k_x, md	300	
k_y, md	300	
k_z, md	100	

Figures 5.1 and 5.2 show how varying observation well length affects the flux distribution in the active and observation wells respectively.

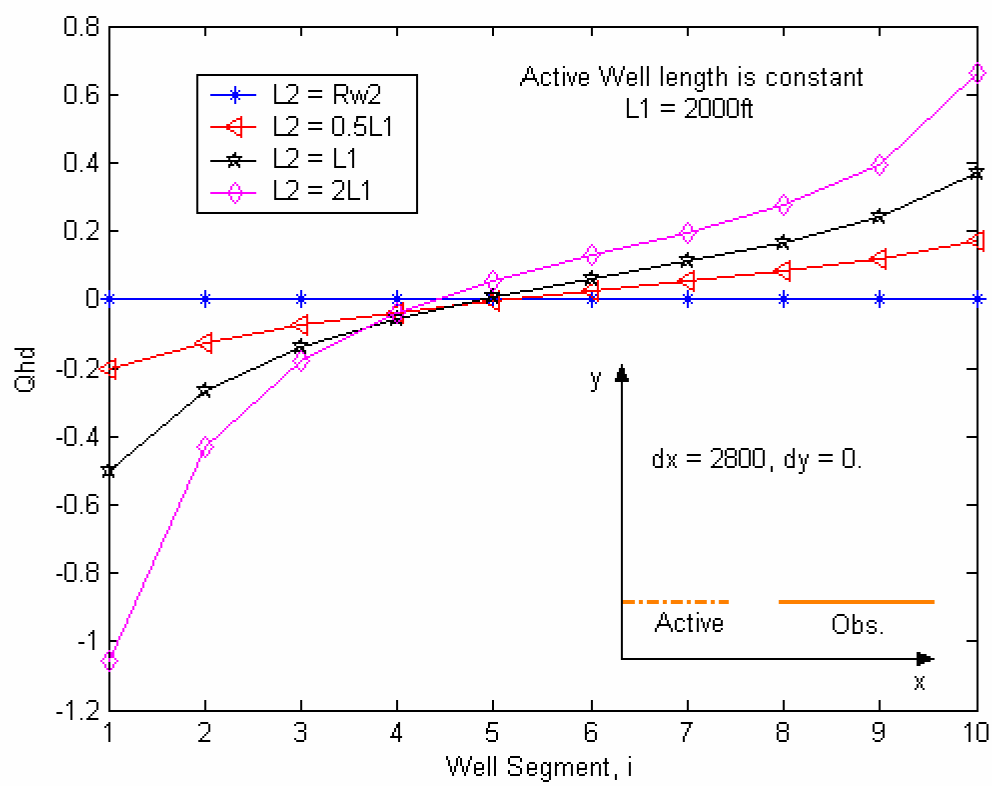

Figure 5.1 Effect of Observation Well Length on Flux Distribution in the Observation Well (Offset in x-direction)

At late time there is increase in flux into and out of the observation well as the length increases. That is flux increases with observation well length. For the configuration of wells presented in Figure 5.1, the flux enters the horizontal observation well at the far end and exits it at the near end. When the observation well length is however made equal to its radius no flux is observed in it as expected. Generally, when the observation well length is made very small (almost zero), no flux enters or leaves it. That is, a vertical observation well will not have any flux in it due to production from an active well.

Figure 5.2 depicts the effect of observation well length on flux distribution in the active well.

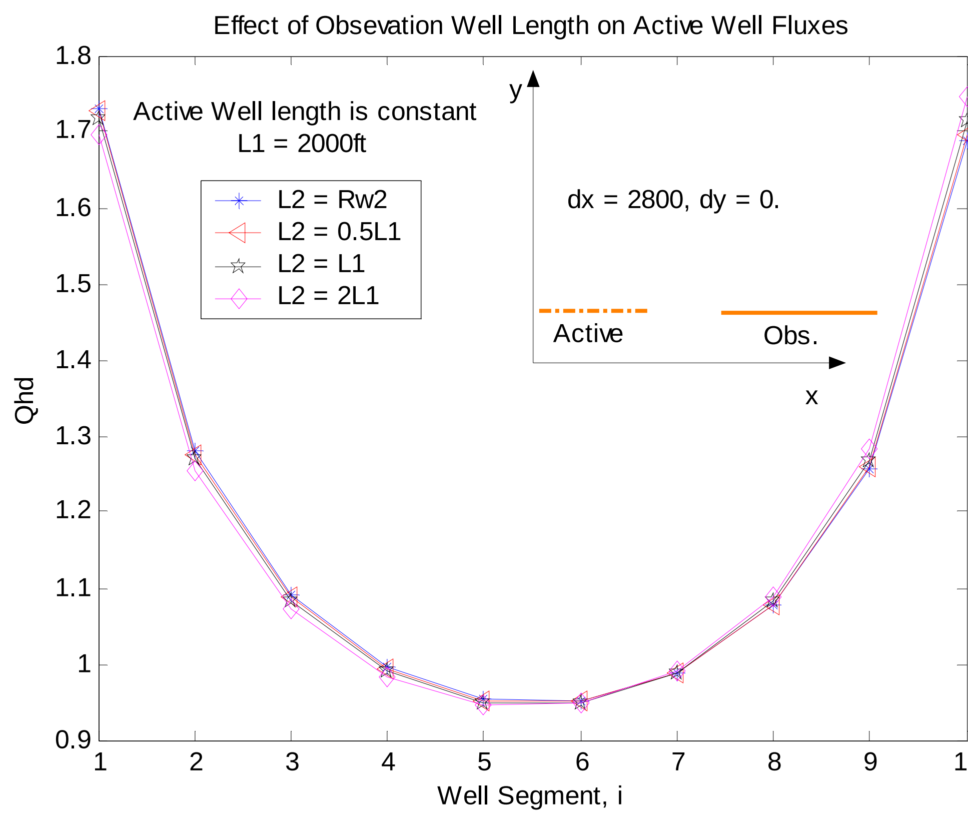

Figure 5.2 Effect of Observation Well Length on Flux Distribution in the Active Well

(Offset in x-direction)

Again it has been shown [19] that at early time the flux distribution in the active well is not at all affected by the observation well because it takes some time for the pressure disturbance created by the active well to reach the observation well. At late time however, the observation well length has some effect, though not significant.

We further present a second configuration of wells in the reservoir. In this second case, the wells have their wellbore located on the same x-coordinate but have an offset of 2000 ft between them in the y-direction. The data used for this well configuration are presented in Table 5.2.

Table 5.2- Data Used to Study Flux Distribution (Offset in y -direction)

Well parameters	Well 1	Well 2
$L,\ ft$	2000	0.165, 1000, 2000, 4000
$r_w,\ ft$	0.165	0.165
$x_w,\ ft$	0.000	0
$y_w,\ ft$	0.000	2000
$z_w,\ ft$	50	50
N_{Re_t}	23889.9	-
Reservoir Parameters		
$h,\ ft$	100	
k_x, md	300	
k_y, md	300	
k_z, md	100	

The configuration of wells and results for the data shown on Table 5.2 is presented in Figures 5.3 and 5.4.

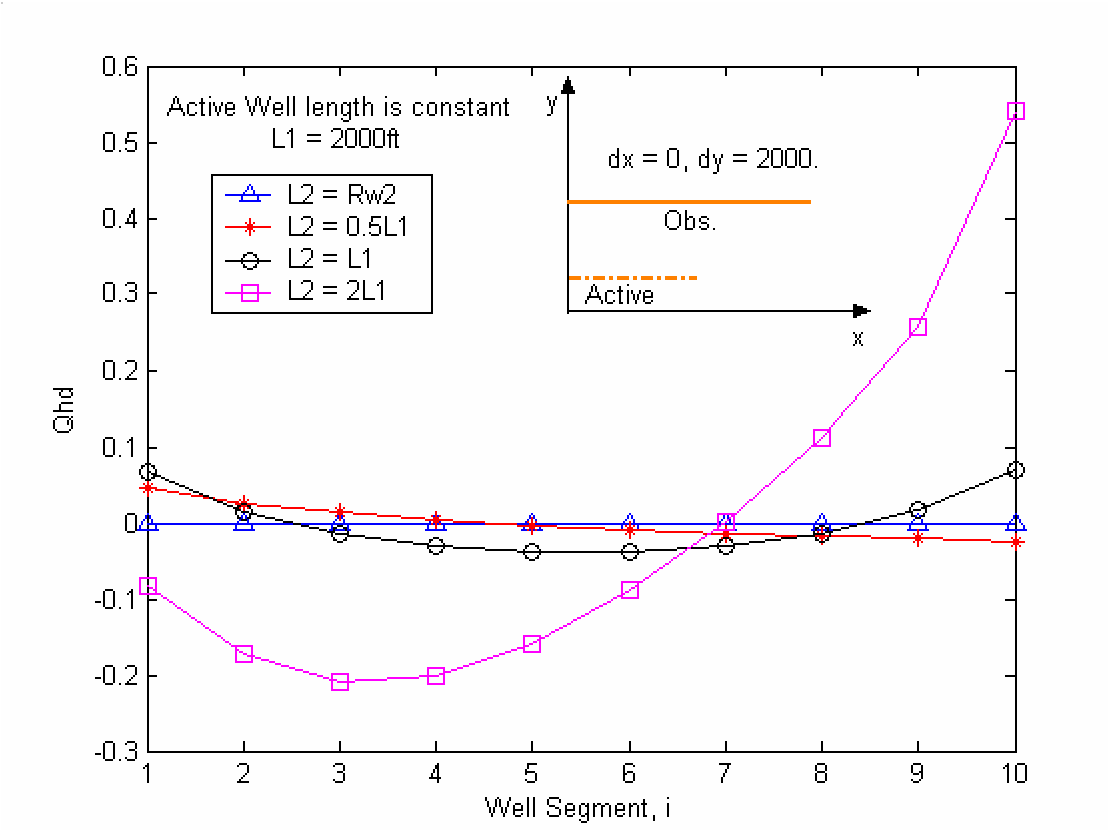

Figure 5.3 Effect of Observation Well Length on Flux Distribution in the Observation Well (Offset in y-direction)

In Figure 5.3, it is seen that the well length has a major control on flux distribution in the observation well. When the observation well length is smaller than the active well length, flux enters the observation well through its left end and exits it through the right end. This is because, for this situation, the active well length extends beyond the observation well length on the right side and creates more fluid flow at this end. When the observation well length becomes greater than the active well length, the reverse is seen apparently because the extra length of the observation well on the right side is little affected by the pressure disturbance at the active well. When both wells are of equal length, we have a U-shaped flux distribution.

Again, the effect of observation well length on the flux distribution in the active well is insignificant for this configuration of wells as can be seen from Figure 5.4.

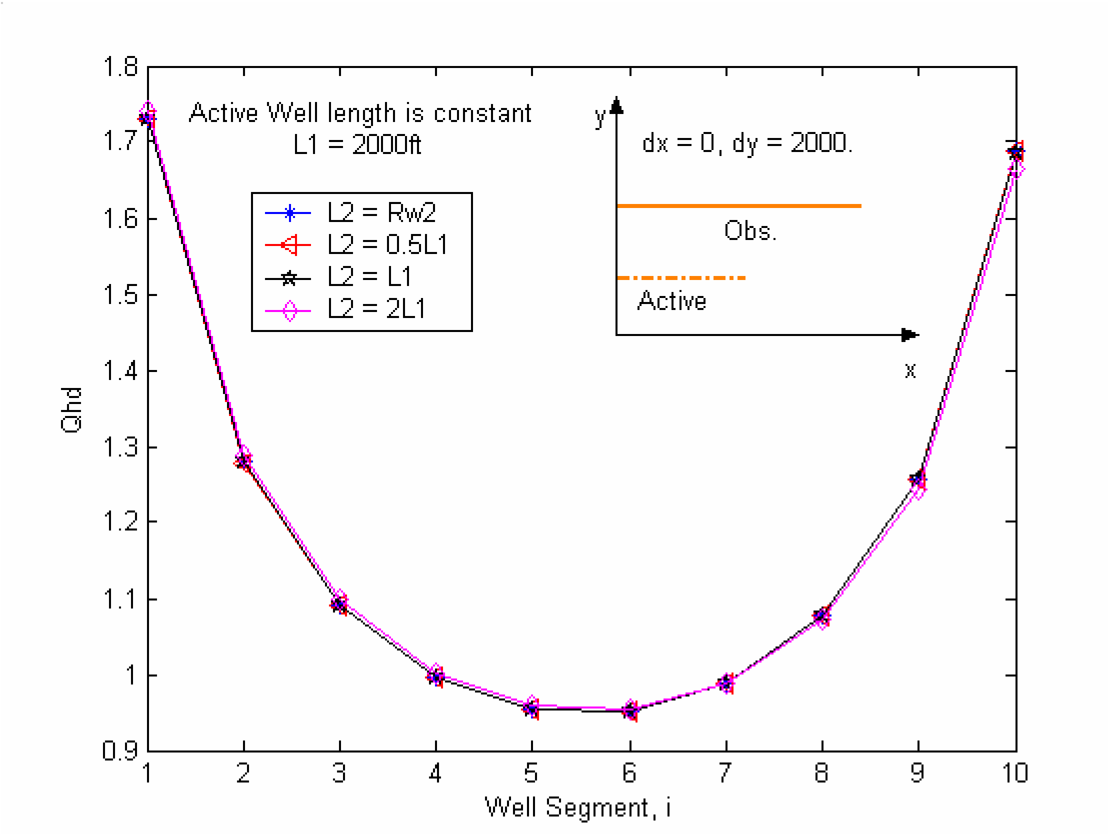

Figure 5.4 Effect of Observation Well Length on Flux Distribution in the Active Well

(Offset in y-direction)

Finally we studied the effect of varying observation well length on flux distribution in both wells when the wells are centered. Here, the centers of both wells are on the same coordinate point in the x-direction. Their centers are fixed in spite of varying the length of the observation well.

Table 5.3- Data Used to Study Flux Distribution (Wells are symmetrically aligned)

Well parameters	Well 1	Well 2
$L,\ ft$	2000	0.165, 1000, 2000, 3000, 4000
$r_w,\ ft$	0.165	0.165
$x_w,\ ft$	0.000	1000, 500, 0, -500, -1000
$y_w,\ ft$	0.000	2000
$z_w,\ ft$	50	50
N_{Re_t}	23889.9	-
Reservoir Parameters		
$h,\ ft$	100	
k_x, md	300	
k_y, md	300	
k_z, md	100	

The results for these cases are presented in Figures 5.5 and 5.6.

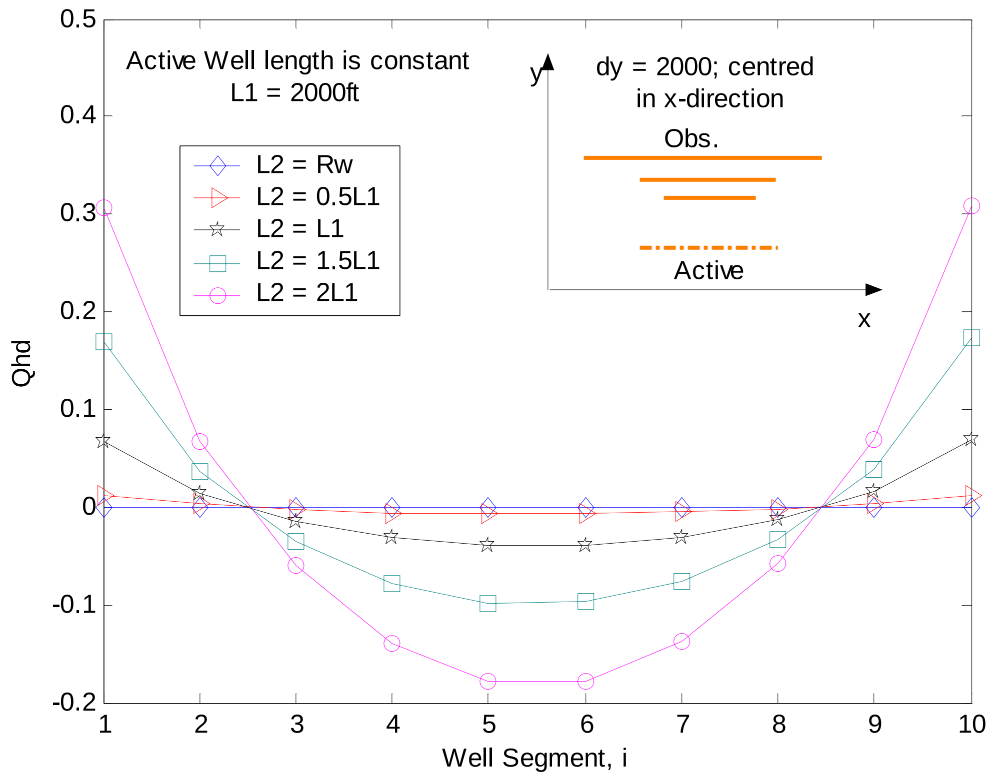

Figure 5.5 Effect of Observation Well Length on Flux Distribution in the Observation Well (Wells are symmetrically aligned)

Figure 5.5 shows that a vertical observation well has no flux moving in or out of it. This is noticed in all other well configurations studied. Flux increases with increasing observation well length. Since both wells are centrally aligned, the flux distribution is U-shaped for all observation well lengths considered.

Figure 5.6 shows that there is virtually no difference in flux distribution in the active well for different observation well lengths when the wells are centrally aligned.

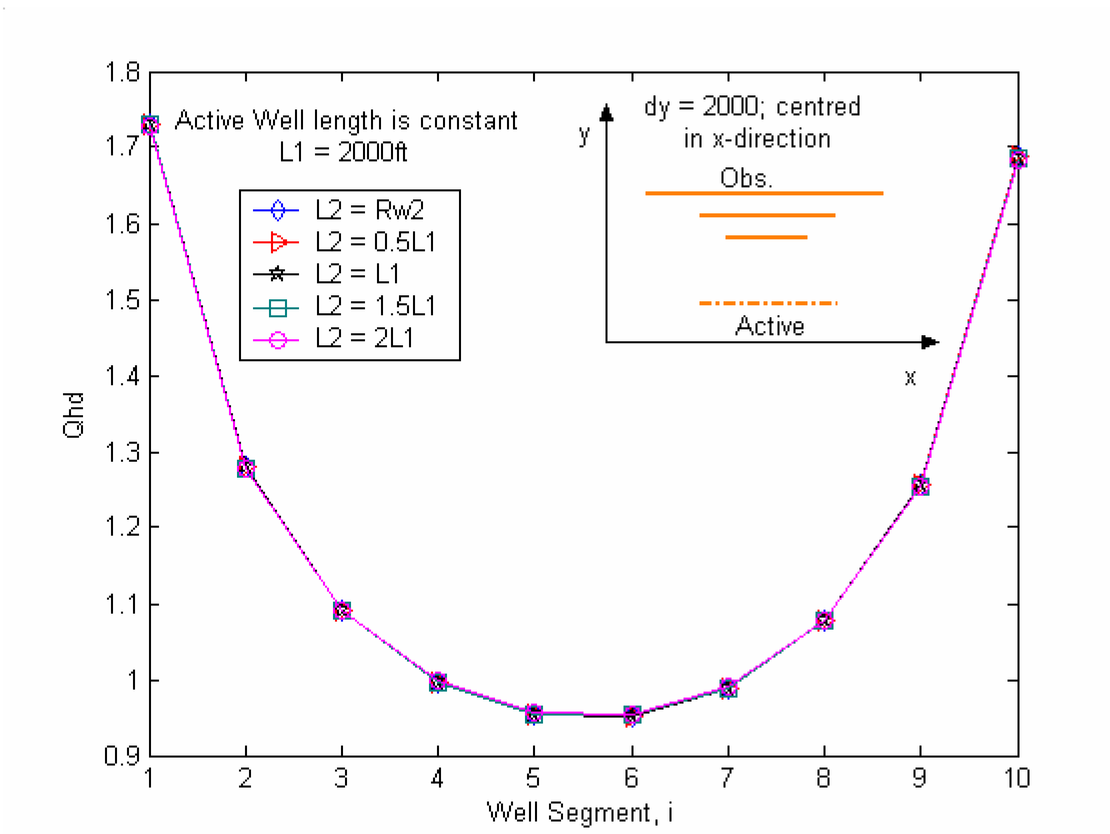

Figure 5.6 Effect of Observation Well Length on Flux Distribution in the Active Well

(Wells are symmetrically aligned)

5.2 Pressure Response

Here we study the effect of changing well lengths on the transient pressure response in both wells. The wellbore pressure is measured at the heels of the two wells. The data used for the three configurations of wells presented here are the same as presented already in Tables 5.1, 5.2, and 5.3 respectively.

5.2.1 Effect of Changing Observation Well Length

In order to study the pressure responses in both wells to varying length of observation well, we kept the active well length constant and vary the observation well length. These responses are shown in Figures 5.7 to 5.9 below.

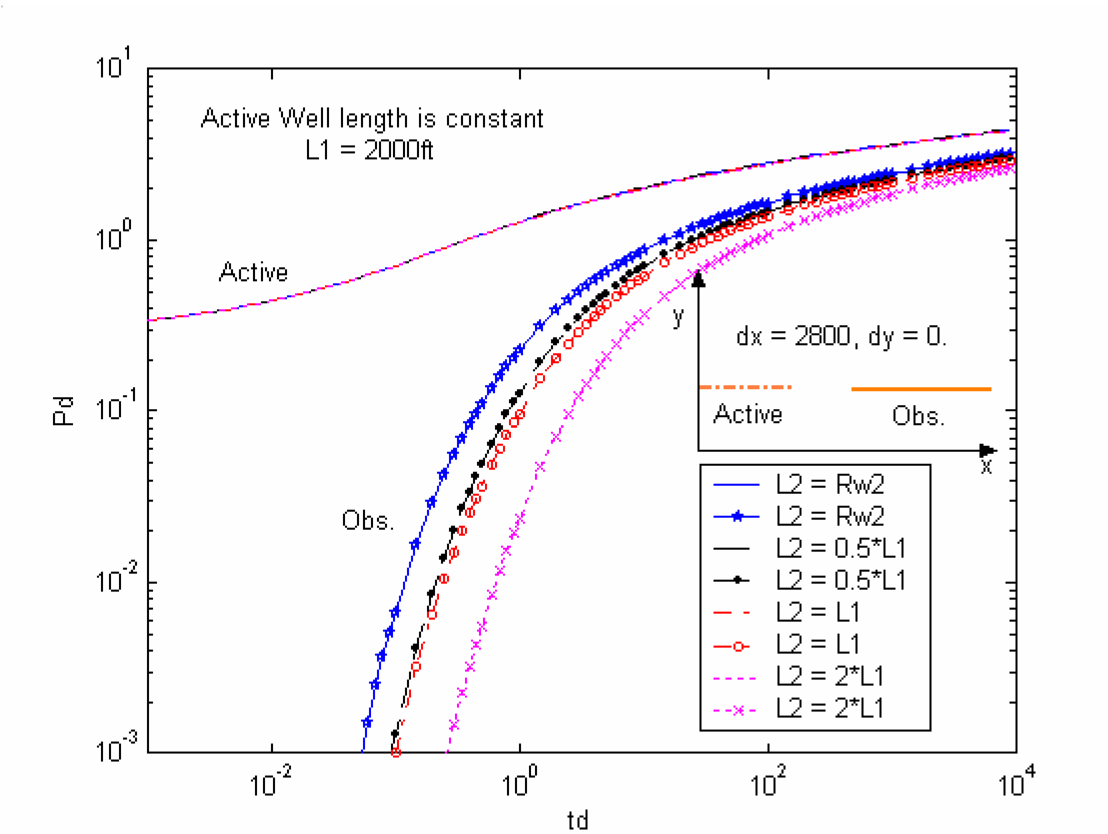

Figure 5.7 Effect of Observation Well Length on Active and Observation Well Pressure Responses (Offset in x-direction)

It can be seen from the result presented in Figure 5.7 that no effect is noticed on the pressure response in the active well by changing the horizontal observation well length especially at early times. At late times, some effects, though insignificant is seen on the pressure response in the active well. This is primarily due to the flux that occurs in the observation well at late times.

However the observation well length has considerable effect on the pressure response at its heel. From Figure 5.7, it is seen that replacing the horizontal observation well by a vertical observation well placed at its heel will significantly increase the pressure response measured at this point. The pressure response at the heel of the horizontal observation well decreases as the observation well length increases. For the well configuration presented in Figure 5.7, it can be seen that the observation well pressure response decreases gradually as its length increases up to the magnitude of the active well length. As the observation well length becomes greater than the active well length, there is more drastic reduction in pressure response at the observation well as observation well length increases.

We now consider a second configuration of well as shown in Figure 5.8 below.

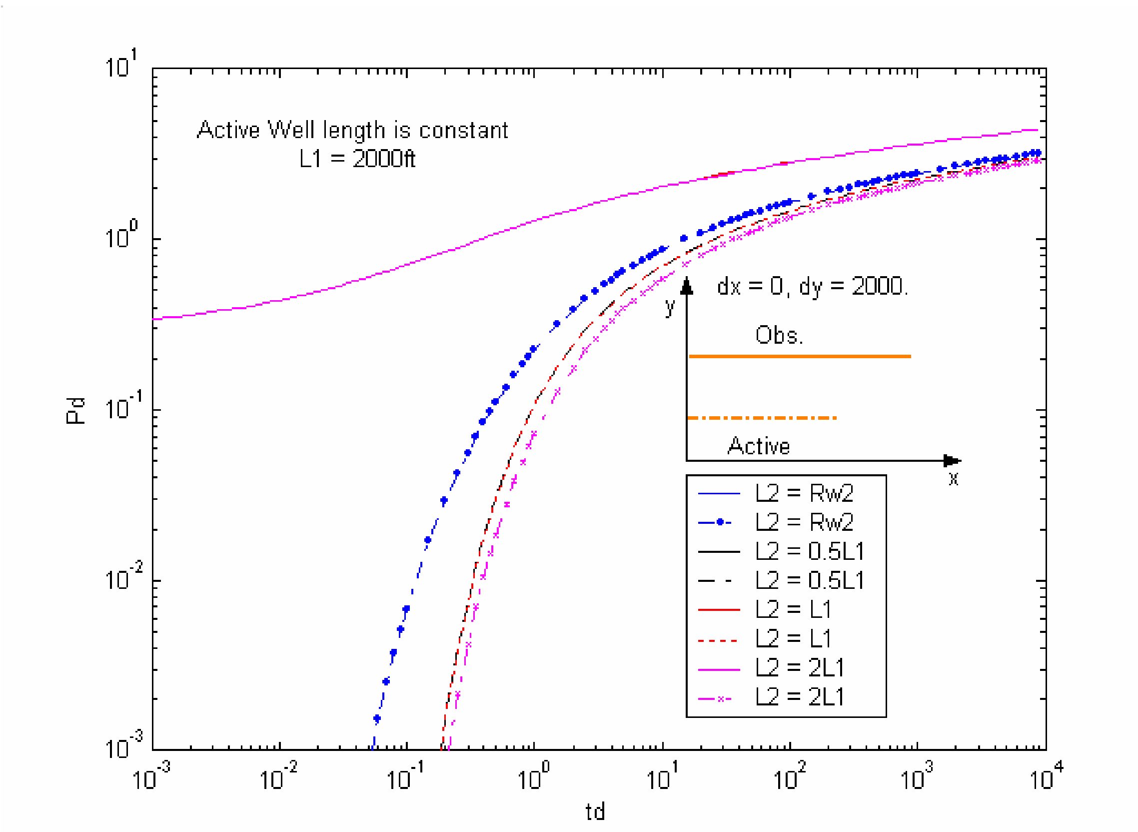

Figure 5.8 Effect of Observation Well Length on Active and Observation Well Pressure Responses (Offset in y-direction)

In Figure 5.8, the active well pressure response does not vary with observation well length. This is the same observation seen in the previous well configuration considered earlier. Observation well pressure response shows a declining trend as observation well length increases.

Finally, we consider the well configuration shown in Figure 5.9.

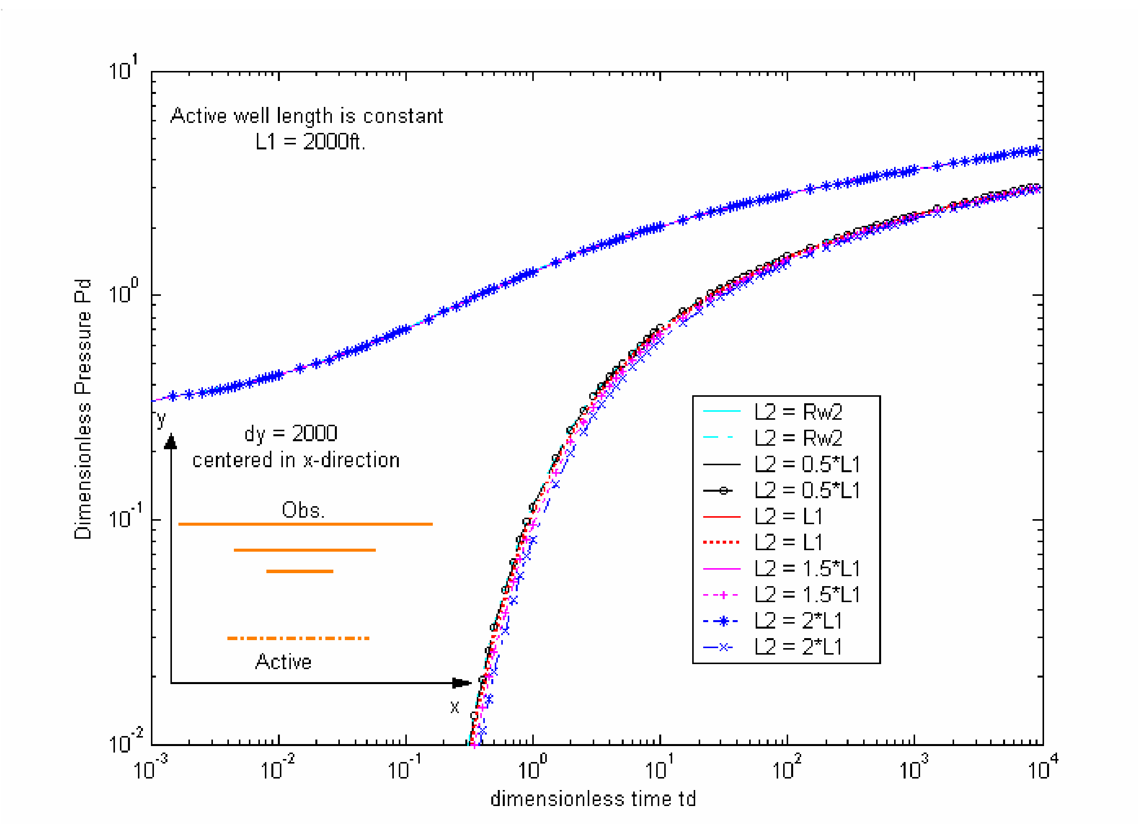

Figure 5.9 Effect of Observation Well Length on Active and Observation Well Pressure Responses (Wells symmetrically aligned)

Again, in Figure 5.9, we observe that the active well pressure response is not affected by the length of the observation well. This is the same as was observed in the other two previous well configurations. However, in this case, the variation in observation pressure with length is very minimal. That is, when the distance between the centers of active and observation wells is kept constant and the length of the observation well is varied, there is minimal variation in observation well pressure but no variation in active well pressure response. This may be in response to changes in horizontal separation between the two well, in addition to changes in well length.

5.2.2 Effect of Changing Active Well Length

We also investigated the responses at both wells to changing active well length. Because we have used the active well half length as the characteristic length, we could only make valid comparison using real pressure responses instead of dimensionless pressure responses as used in the comparisons above. We have chosen a single configuration of wells in this case, as we expect other configurations to give similar trends, to illustrate the results obtained from this study. The data for this configuration of wells is presented in Table 5.4.

Table 5.4- Data Used to Study Pressure Response to Active Well Length

Well parameters	Well 1	Well 2
$L,\ ft$	1000, 2000, 4000	1000
$r_w,\ ft$	0.165	0.165
$x_w,\ ft$	0.000	2800
$y_w,\ ft$	0.000	2000
$z_w,\ ft$	50	50
N_{Re_t}	23889.9	-
Reservoir Parameters		
$h,\ ft$	100	
k_x, md	200	
k_y, md	300	
k_z, md	100	

Figure 5.10 presents the responses obtained from active and observation wells when the active well length is varied while keeping the observation well length constant. We observe that the active well pressure response declines with increasing length of active well but the observation well pressure response increases with increasing active well length. Both observations are rational and expected. We expect the pressure drop at the active well to reduce as more area become open to flow, i.e. increased active well length. However, an increased flow to the active well is expected to cause significant decline in pressure at some distant location in the reservoir, in this case, the observation well location.

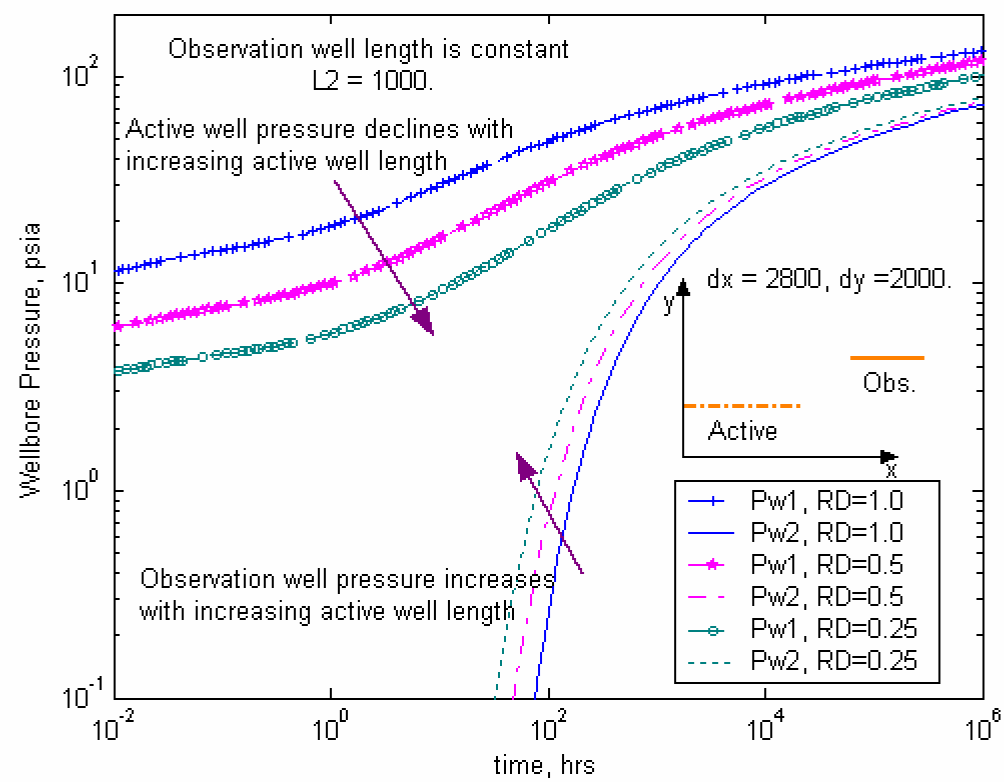

Figure 5.10 Effect of Active Well Length on Active and Observation Well Pressure Responses (Offset in x and y-direction)

5.2.3 Effect of Changing Active and Observation Well Lengths

Here we study the effects of varying both well lengths on pressure response. For each case studied, we keep the ratio $R_D = \frac{L_2}{L_1}$ constant and see how pressure response changes at both wells. Because our model uses the half length of the active well as the characteristic length we can not compare the pressure responses in dimensionless form when active well length changes. Therefore all comparisons here are presented in terms of actual pressure responses. All well and reservoir parameters are held constant while lengths are varied appropriately. Four ratios 0.5, 1.0, 1.5, 2.0 were chosen in this study and the results are presented below.

5.2.3.1 Case 1A: Well Ratio Equals 1 $R_D = 1.0$

For the case in which the well ratio is one, we increased the length of both wells from 1000 ft to 2000 ft and then to 4000 ft to study the effect of changing well lengths on active and observation well responses. The data used for Case 1 are presented in Table 5.5 while the results are presented in Figure 5.11.

Table 5.5- Data Used to Study Pressure Response in Case 1A

Well parameters	Well 1	Well 2
$L,\ ft$	1000, 2000, 4000	1000, 2000, 4000
$r_w,\ ft$	0.165	0.165
$x_w,\ ft$	0.000	2800
$y_w,\ ft$	0.000	2000
$z_w,\ ft$	50	50
N_{Re_t}	23889.9	-
Reservoir Parameters		
$h,\ ft$	100	
k_x, md	200	
k_y, md	300	
k_z, md	100	

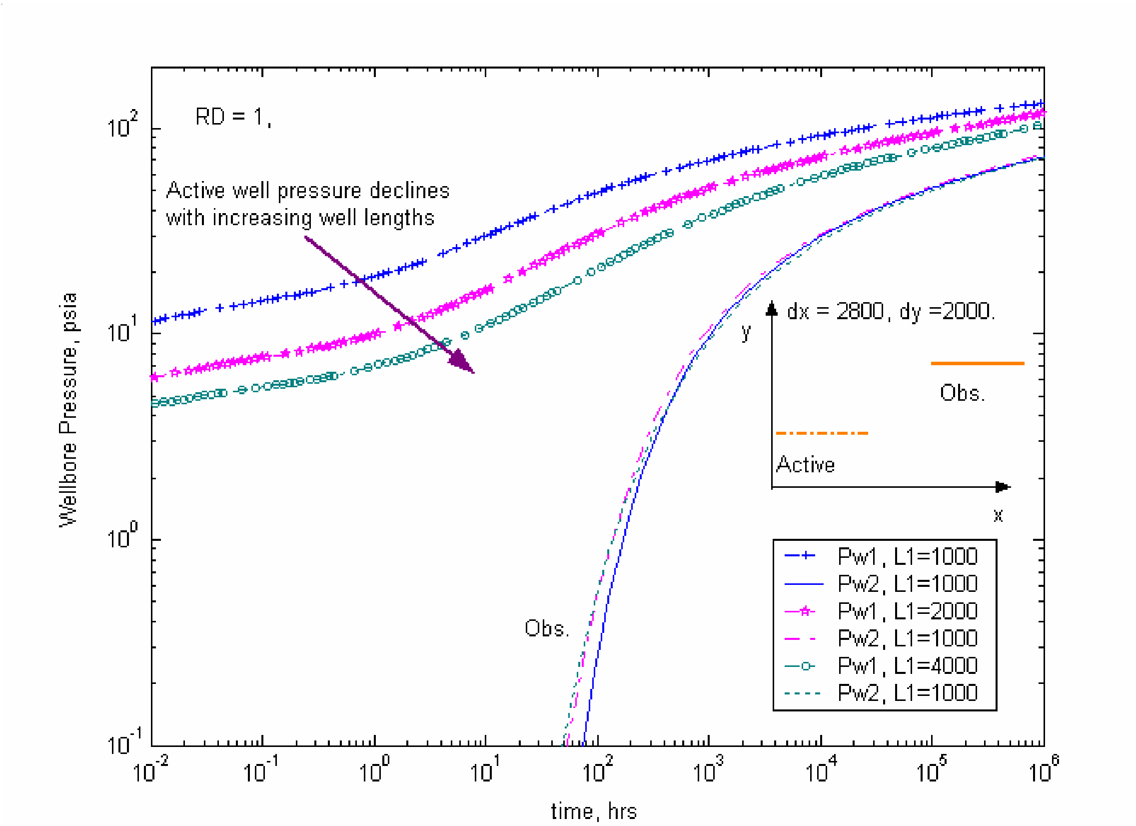

Figure-5.11 Effect of Varying the Length of both Wells on Active and Observation Well Pressure Responses ($R_D = 1.0$)

Figure 5.11 shows the variation of active and observation well pressure responses with changing lengths. We observe that as both lengths increase the active well pressure decreases while the observation well pressure increases slightly. This simply shows, as anticipated, that the effect of increasing active well length has overshadowed that of increasing observation well length. As observed in earlier sections of this work, increasing observation well length will cause a decline in observation well response. Our observation here is however contrary to this because both well lengths are increased simultaneously. Therefore, even though the increase in observation well length has caused some decrease in its pressure response, corresponding increase in the active well response has caused a larger rise in observation well response so that the net effect seen is a slight increase in observation well pressure response. Figure 5.11 further shows that an increase in the length of both wells will cause considerable decline in active well pressure response. This is expected because the greater flow area presented by a longer active well length will cause a reduction in reservoir pressure drop as seen at the active wellbore. Although a longer active well length will cause additional pressure drop due to friction, this additional pressure drop is relatively small compared to the pressure drop due to transient flow in the reservoir. We have also shown earlier that an increase in observation well length has no effect, worth considering, on active well pressure response.

5.2.3.2 Case 2A: Well Ratio Equals Half $R_D = 0.5$

Here we present, for the same well configuration presented above, a case where the observation well length is always half of the active well length. Table 5.6 shows the data used for Case 2A.

Table 5.6- Data Used to Study Pressure Response in Case 2A

Well parameters	Well 1	Well 2
$L,\ ft$	1000, 2000, 3000	500, 1000, 1500
$r_w,\ ft$	0.165	0.165
$x_w,\ ft$	0.000	2800
$y_w,\ ft$	0.000	2000
$z_w,\ ft$	50	50
N_{Re_t}	23889.9	-
$q, stb/day$	2000	0

Reservoir Parameters		Fluid Properties	
$h,\ ft$	100	μ, cp	1.5
k_x, md	200	$B, rb/stb$	1.1
k_y, md	300	c_t	3.5E-6
k_z, md	100		
ϕ	0.25		

Results for this Case 2A are presented in Figure 5.12.

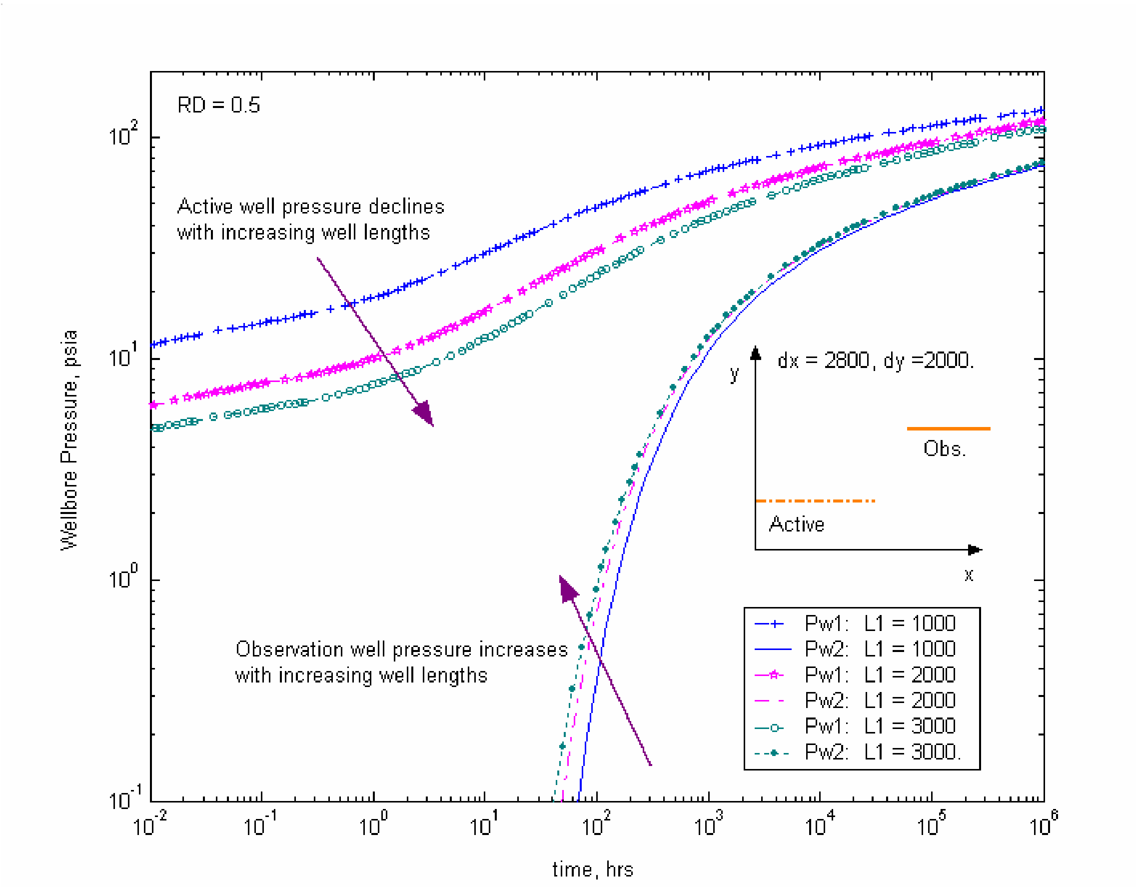

Figure 5.12 Effect of Varying the Length of both Wells on Active and Observation Well Pressure Responses ($R_D = 0.5$)

Figure 5.12 shows a trend similar to that observed in Figure 5.11. Again, it is obvious here that the effect of increasing the active well length has slightly overridden that of increasing the observation well length so that observation well pressure increases slightly with increasing well lengths.

5.2.3.3 Case 3A: Well Ratio Equals One and Half $R_D = 1.5$

In this case, the observation well length is one-and-half times the active well length.

Table 5.7- Data Used to Study Pressure Response in Case 3A

<table>
<tr><td colspan="2">Well parameters</td><td colspan="2">Well 1</td><td colspan="2">Well 2</td></tr>
<tr><td colspan="2">$L,\ ft$</td><td colspan="2">500, 1000, 2000</td><td colspan="2">750, 1500, 3000</td></tr>
<tr><td colspan="2">$r_w,\ ft$</td><td colspan="2">0.165</td><td colspan="2">0.165</td></tr>
<tr><td colspan="2">$x_w,\ ft$</td><td colspan="2">0.000</td><td colspan="2">2800</td></tr>
<tr><td colspan="2">$y_w,\ ft$</td><td colspan="2">0.000</td><td colspan="2">2000</td></tr>
<tr><td colspan="2">$z_w,\ ft$</td><td colspan="2">50</td><td colspan="2">50</td></tr>
<tr><td colspan="2">N_{Re_t}</td><td colspan="2">23889.9</td><td colspan="2">-</td></tr>
<tr><td colspan="2">$q,\ stb/day$</td><td colspan="2">2000</td><td colspan="2">0</td></tr>
<tr><td colspan="3">Reservoir Parameters</td><td colspan="3">Fluid Properties</td></tr>
<tr><td>$h,\ ft$</td><td colspan="2">100</td><td colspan="2">$\mu,\ cp$</td><td>1.5</td></tr>
<tr><td>$k_x,\ md$</td><td colspan="2">200</td><td colspan="2">$B,\ rb/stb$</td><td>1.1</td></tr>
<tr><td>$k_y,\ md$</td><td colspan="2">300</td><td colspan="2">c_t</td><td>3.5E-6</td></tr>
<tr><td>$k_z,\ md$</td><td colspan="2">100</td><td colspan="3"></td></tr>
<tr><td>ϕ</td><td colspan="2">0.25</td><td colspan="3"></td></tr>
</table>

Results obtained from Case 3A show an observation pressure trend that is different from those obtained from Cases 1A and 2A considered above. This is illustrated in Figure 5.13.

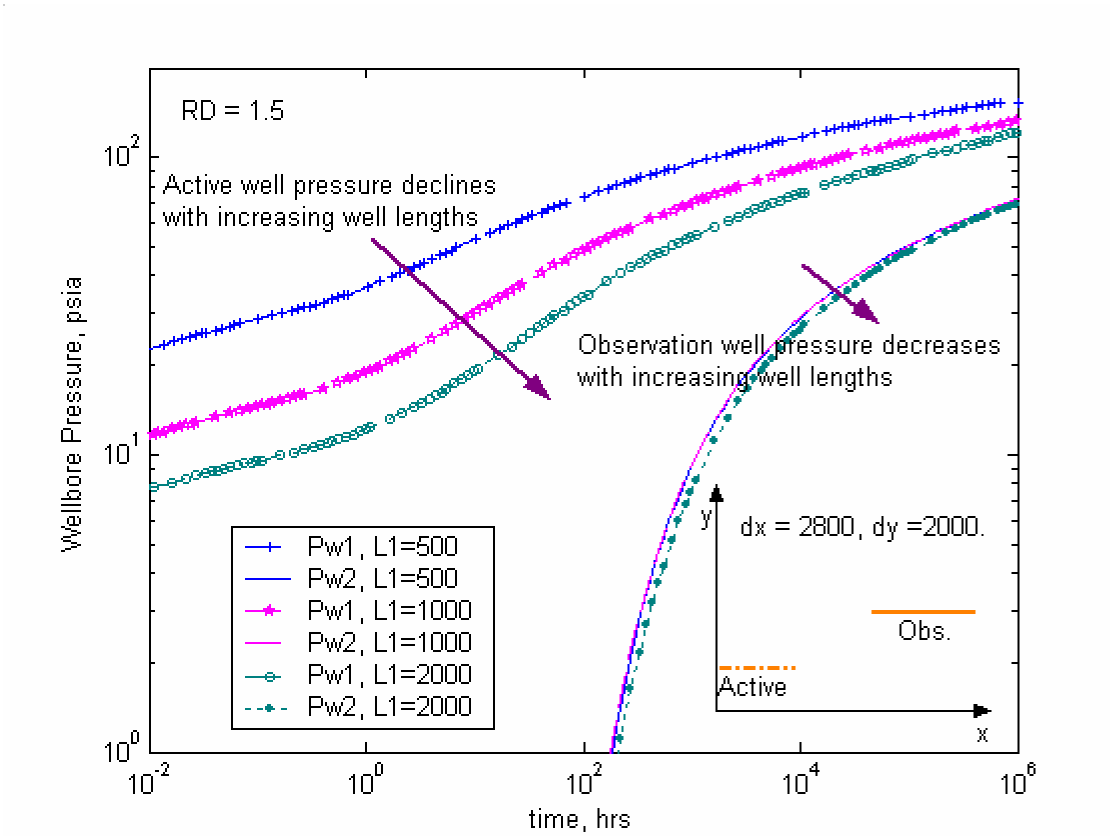

Figure 5.13 Effect of Varying the Length of both Wells on Active and Observation Well Pressure Responses ($R_D = 1.5$)

In Figure 5.13 the active well pressure response declines as both well lengths are increased. Increasing active well length from 500 ft to 1000 ft, with a proportionate increase in observation well length, does not produce any appreciable change in observation well pressure response. When the active well length is doubled to 2000 ft, again with corresponding increase in observation well length (noting that $R_D = 1.5$), a decline in observation well pressure response is noticed. We may thus conclude that in this case, the effect of increasing the active well length is not significant enough to override that of increasing the observation well length. We have noted earlier that the increasing the observation well length will induce a decrease in observation well pressure response. Therefore, we may conclude once more, that the effect of increasing active well length on observation well pressure response is opposite to that of increasing the observation well length and that the pressure response observed (at the observation well) is a combined effect of both.

We now increase the active well length to 4000 ft in Case 3A to further study the trend of the pressure response. The result of this is shown in Figure 5.14.

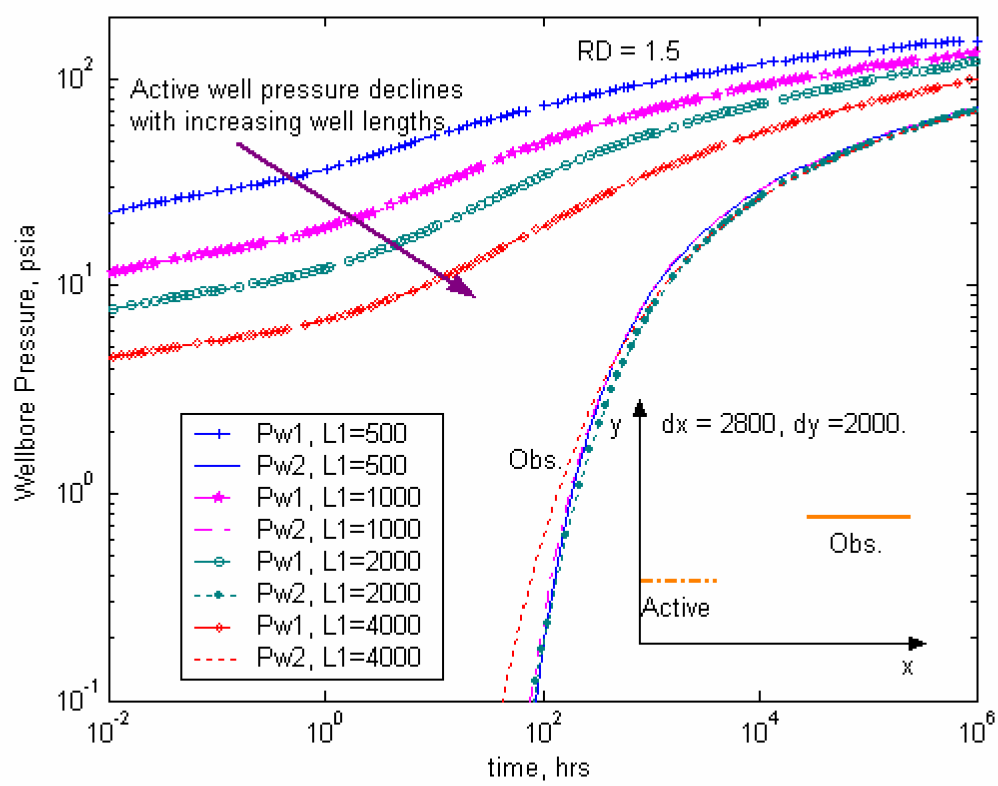

Figure 5.14 Effect of Varying the Length of both Wells on Active and Observation Well Pressure Responses ($R_D = 1.5$)

In Figure 5.14 we observe that when the active well length is increased from 2000 ft to 4000 ft, with corresponding increase in observation well length (i.e. observation well length is increased to 6000 ft), the observation well pressure response becomes slightly greater than the response seen with shorter active well lengths, especially at intermediate time. This observation again suggests that for any particular well ratio, $R_D = \frac{L_2}{L_1}$, observation pressure response from increasing the active well length will at some time override any pressure response due to corresponding increase in observation well length so that the observed response at the observation well begin to rise with this increase. This may be due to a change in the relative geometry of the wells and an overlap along the x-axis.

5.2.3.4 Case 4A: Well Ratio Equals Two $R_D = 2.0$

Our Observation here is similar to Case 3A. Data for this case are presented in Table 5.8.

Table 5.8- Data Used to Study Pressure Response in Case 4A

Well parameters	Well 1	Well 2
$L,\ ft$	750, 1000, 2000	1500, 2000, 4000
$r_w,\ ft$	0.165	0.165
$x_w,\ ft$	0.000	2800
$y_w,\ ft$	0.000	2000
$z_w,\ ft$	50	50
N_{Re_t}	23889.9	-
$q,\ stb/day$	2000	0

Reservoir Parameters		Fluid Properties	
$h,\ ft$	100	$\mu,\ cp$	1.5
$k_x,\ md$	200	$B,\ rb/stb$	1.1
$k_y,\ md$	300	c_t	3.5E-6
$k_z,\ md$	100		
ϕ	0.25		

Figure 5.15 shows a decline in observation pressure response as active well length increases. Figure 5.15 again shows that when the effect of increasing active well length is not very significant, the effect of increasing observation well length may become visible.

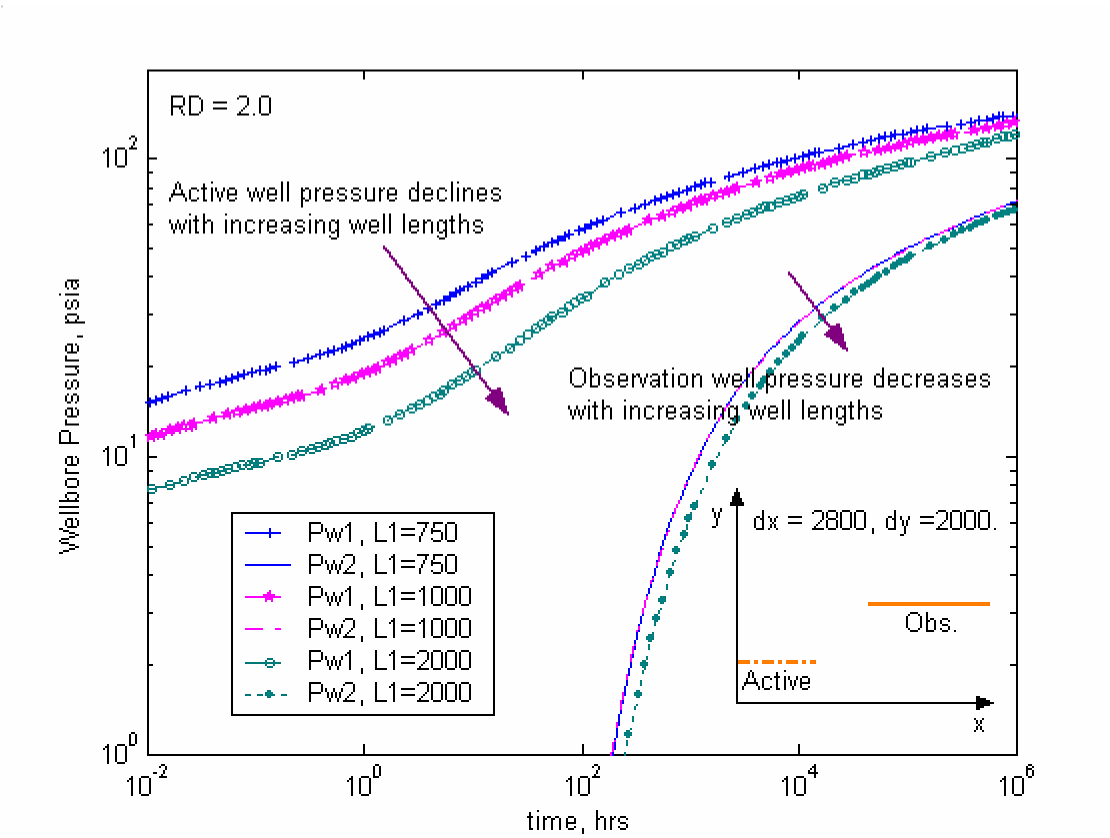

Figure-5.15 Effect of Varying the Length of both Wells on Active and Observation Well Pressure Responses ($R_D = 2.0$)

5.3 Effect of Well Length on Flow Regime

To fully understand and be able to interpret the different signatures encountered in interference test analysis of horizontal wells, we need to know the effects of changing well lengths on the flow regimes identified in pressure signatures. These flow regimes are more clearly identified by semi-log pressure derivative. We thus include in this study the semi-log pressure derivative response to changing lengths. We investigate three cases: flow regime response to changing observation well length, flow regime response to changing active well length, and flow regime response to changing both well lengths. Before considering these cases, it is vital to note that, in the absence of any impairing factors, we do expect to observe the early radial flow, the intermediate linear flow and the late pseudo-radial flow regimes in the active well pressure response. We however do expect the presence of only the late pseudo-radial flow regime in the observation well. This is because it takes some time for the transient disturbance created at the active well to reach the observation well, by which time the early and intermediate time flow regimes may have vanished.

5.3.1 Case 1B: Response to Changing Observation Well Length

Here we investigate the effect of observation well length on the outset of the three major flow regimes encountered in horizontal well analysis. We observe that observation well length has no effect on the early, intermediate and late time flow regimes seen in the active well pressure response. This is expected because observation well length has no effect on active well pressure response at early and intermediate times and an insignificant effect at late times as seen in Figures 5.7 and

5.16. We also observe that increasing observation well length delays the outset of the late pseudo-radial flow regime in the observation well pressure response. In fact, careful observation of Figure 5.16 reveals that the observation well pressure derivative reaches the pseudo-radial flow regime latest for the case in which the observation well length is longest.

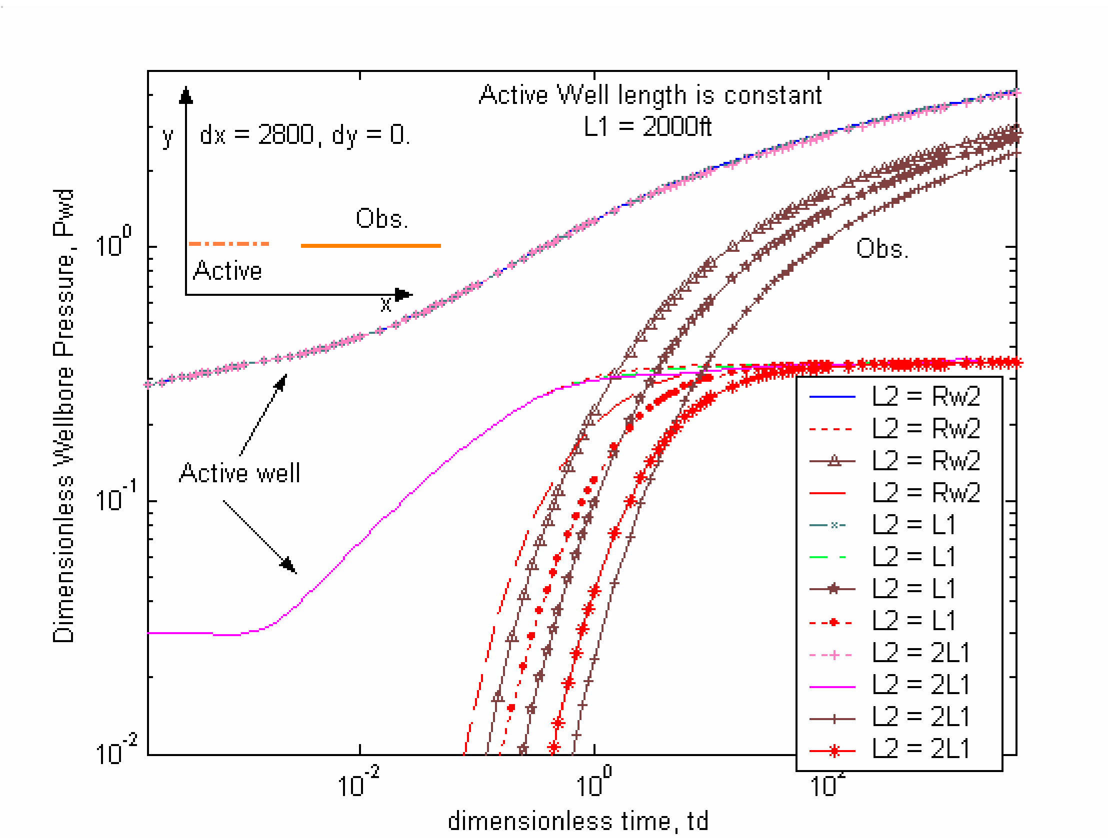

Figure-5.16 Effect of Observation Well Length on Appearance of Flow Regime in Active and Observation Well Pressure Response

5.3.2 Case 2B: Response to Changing Active Well Length

We further study the effect of active well length on flow regime. Figures 5.17, 5.18 and 5.19 show how varying active well length affect the outset of the three major flow regimes.

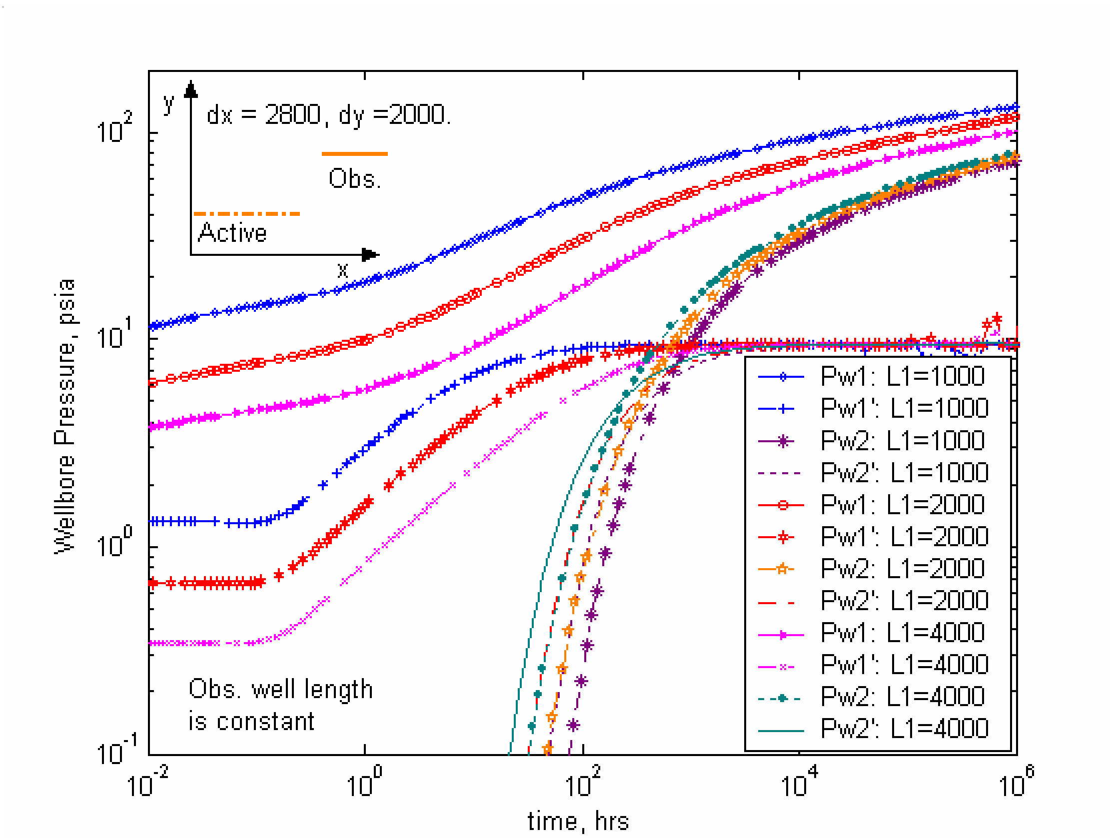

Figure 5.17 Effect of Active Well Length on Appearance of Flow Regime in Active and Observation Well Pressure Response

Figure 5.17 shows that increasing active well length does not hasten or delay the outset of the early and intermediate time flow regimes in the active well pressure response. It however lengthens the duration of the intermediate linear flow regime in the active well. It also delays the outset of the late pseudo-radial flow regime in the active well. Contrarily, increasing active well length hastens the appearance of the late time pseudo-radial flow period in the observation well pressure response.

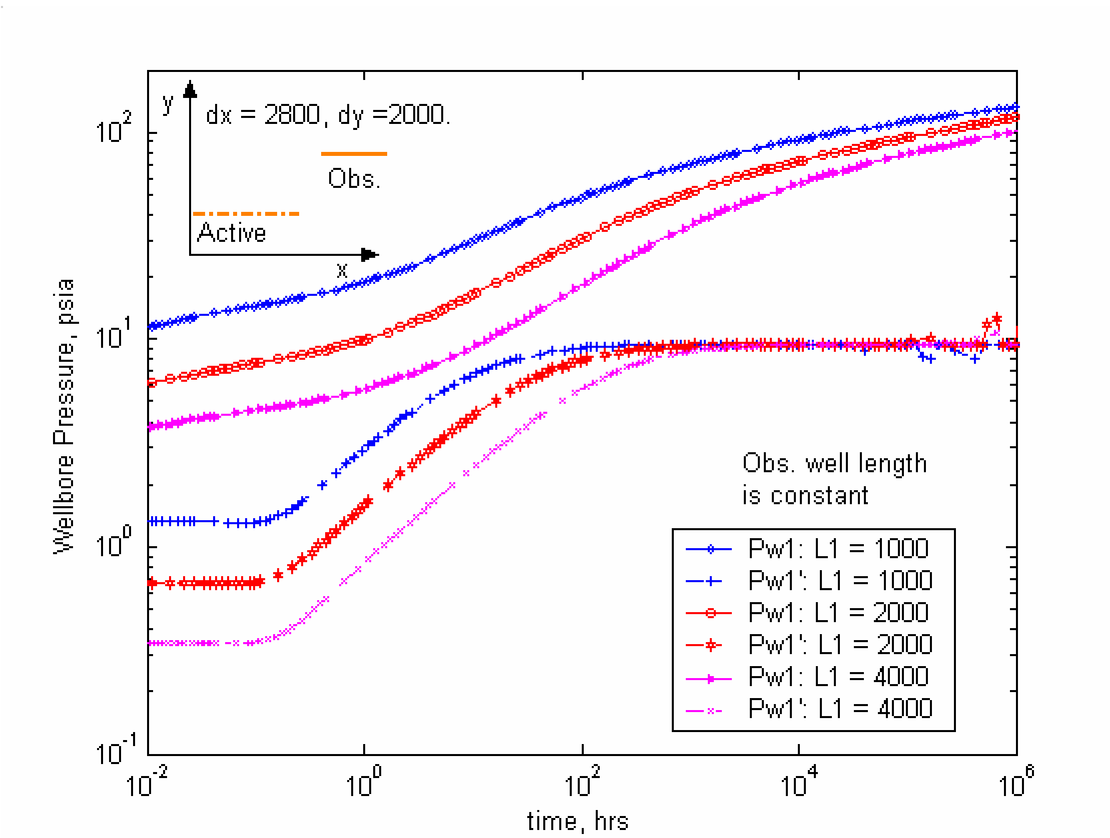

Figure 5.18 Effect of Active Well Length on Appearance of Flow Regime in Active Well Pressure Response

Figure 5.18 presents the same cases presented in Figure 5.17 to illustrate the effect of active well length on the appearance of flow regimes in the active well. We see a marked difference in the outset of the late pseudo-radial flow period for the well lengths considered. We also notice that the intermediate linear flow period begin at the same time irrespective of the active well length. Although not shown here, we know that the early time radial flow period in all the three cases of active well length considered in Figure 5.18 starts exactly at the start of production (some infinitesimally small time).

Again, we present in Figure 5.19 the cases presented earlier in Figure 5.17 to illustrate the effect of active well length on the appearance of flow regime in the observation well.

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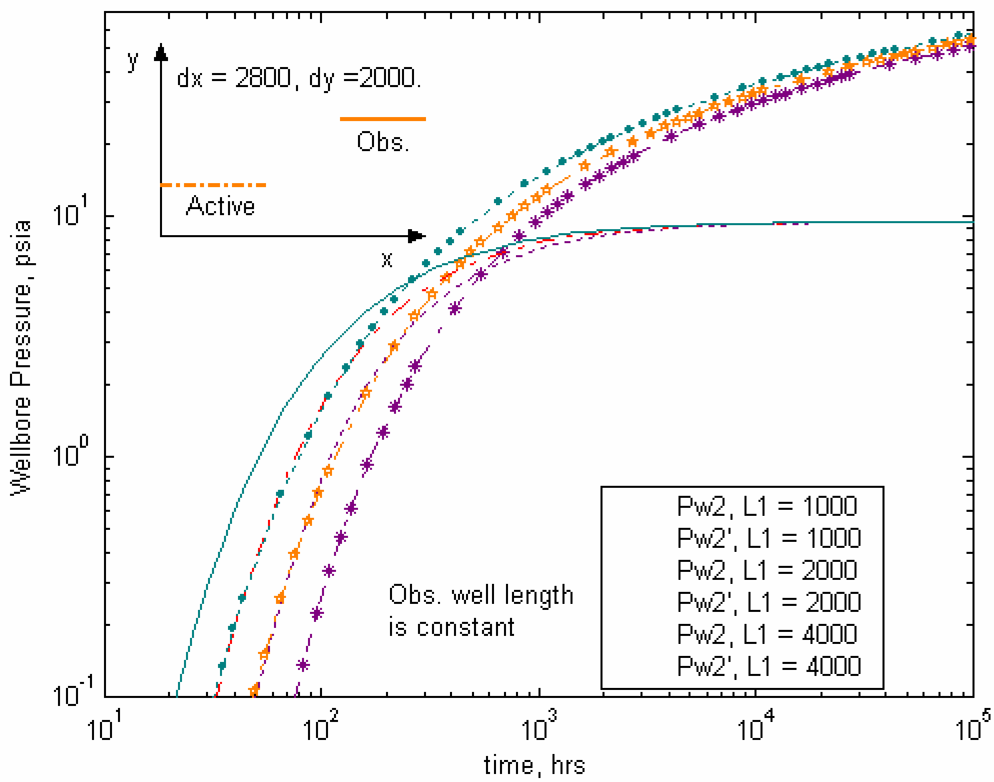

Figure 5.19 Effect of Active Well Length on Appearance of Flow Regime in Observation Well Pressure Response

Figure-5.19 shows that increasing active well length hastens the outset of the late pseudo-radial flow regime.

5.3.3 Case 3B: Response to Changing Active and Observation Well Lengths

Finally, we investigate the effect of changing both well lengths on the appearance of the three main flow regimes. Results are presented in Figures 5.20 to 5.23.

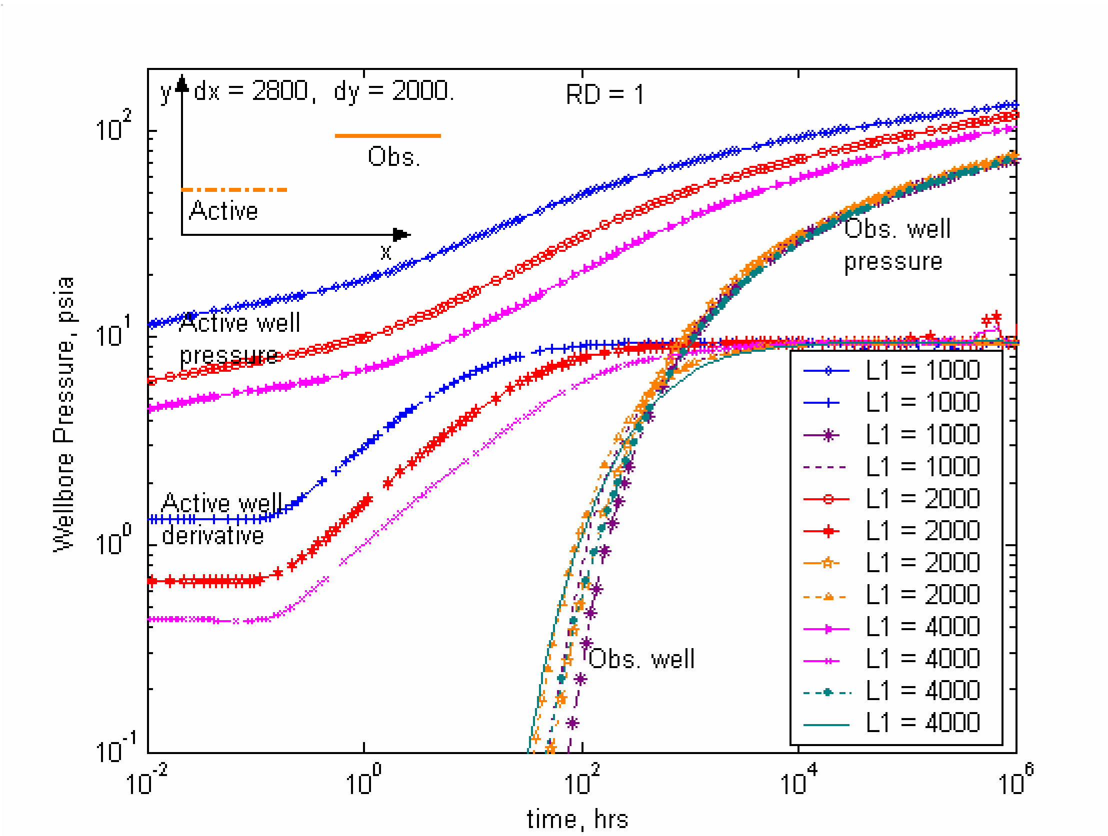

Figure 5.20 Effect of Varying the Lengths of both Wells on Appearance of Flow Regime in Active and Observation Well Pressure Response

Figure 5.20 presents the case in which the wells are of equal length. Trends here are generally similar to that observed when only the active well length is varied except for the observation well pressure regime. It is observed that increasing both well lengths delays the outset of the late time pseudo radial flow regime in the active well pressure response but has no significant effect on the outset of the same flow regime in the observation well pressure response. We have observed from Cases 1B and 2B above that increasing only the active well length (while keeping the observation well length constant) will produce an opposing effect, on the outset of the late time pseudo-radial flow regime, to increasing only the observation well length (while keeping the active well length constant). We may thus conclude that two effects have nullified each other in this case and that effectively there is no effect of increasing both well lengths on the outset of the late pseudo-radial flow regime in the observation well pressure response.

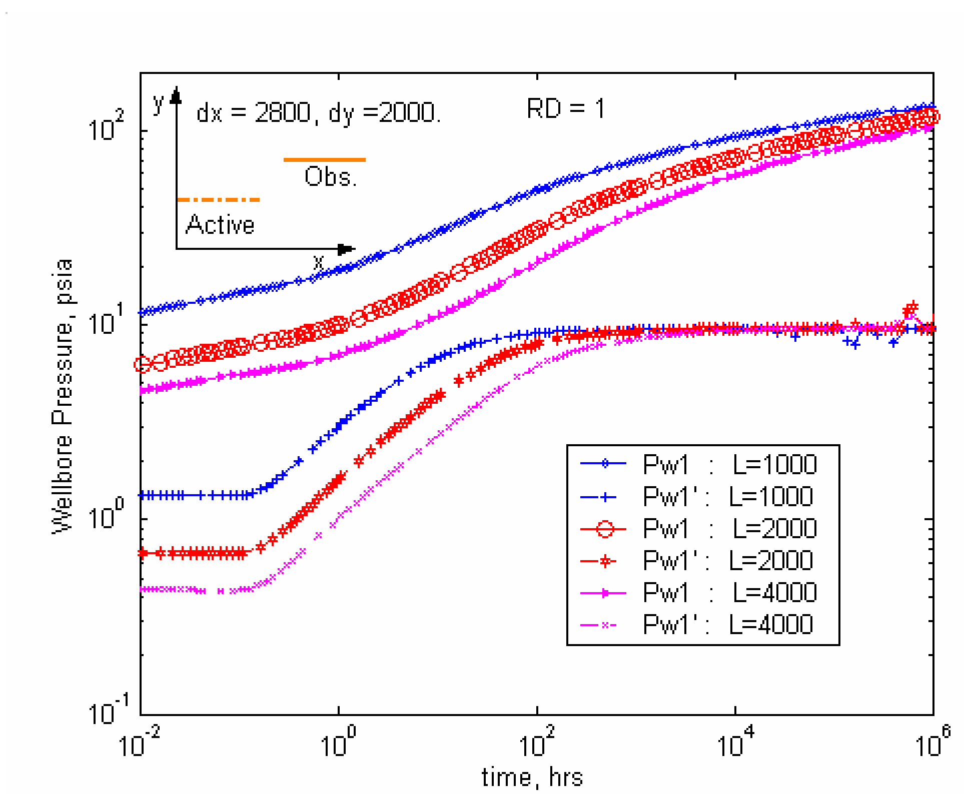

Figure 5.21 Effect of Varying the Lengths of both Wells on Appearance of Flow Regime in Active Well Pressure Response

Figure 5.21 illustrates the delay in the outset of the late pseudo-radial flow regime in the active well pressure response caused by increasing both well lengths simultaneously. The delay in this case is similar to that observed in Case 2B (in which the length of only the active well is increased) because only the increase in active well length contributes to the delay in the pseudo-radial flow regime of the active well pressure response. Figure 5.21 also shows that the outsets of the early and intermediate time flow regimes in the active well pressure response are not affected by variation in the length of both wells.

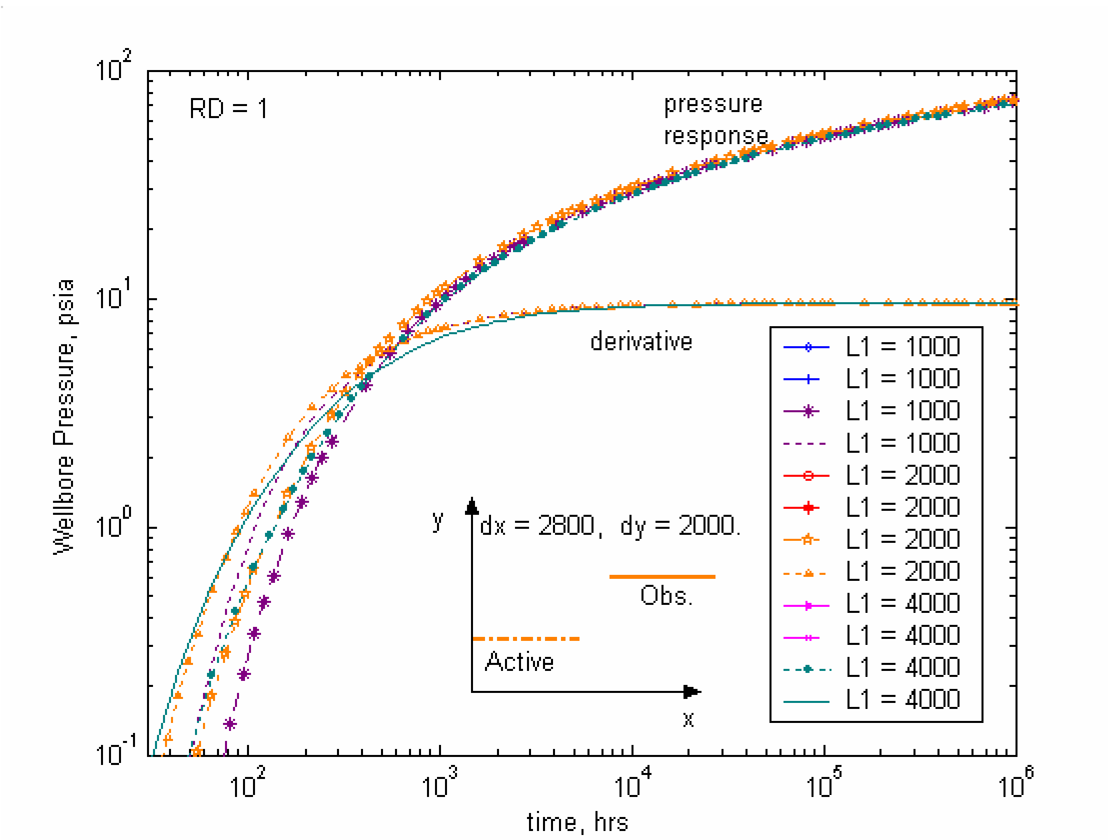

Figure-5.22 Effect of Varying the Lengths of both Wells on Appearance of Flow Regime in Observation Well Pressure Response

Figure 5.22 is presented to show that increasing the length of both wells does not have any significant effect on the appearance of the late time pseudo-radial flow regime in the observation well. In order to study the effect of changing both lengths, on flow regime occurrence when $R_D > 1.0$, we present the case where $R_D = 1.5$ in Figure-5.23 below.

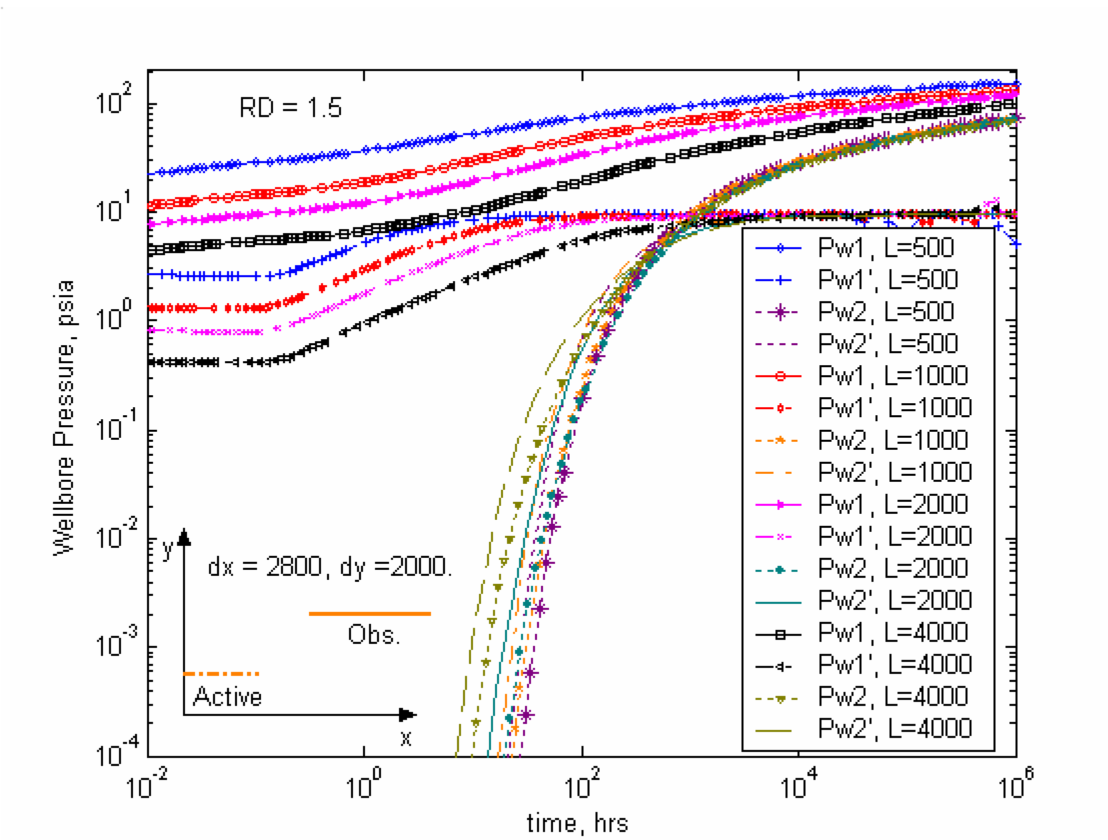

Figure-5.23 Effect of Varying the Lengths of both Wells on Appearance of Flow Regime in Active and Observation Well Pressure Response ($R_D = 1.5$)

Figure 5.23 shows that increasing the lengths of both wells delays the appearance of the late pseudo-radial flow regime in the active well but has no effect on the outset of the late pseudo-radial flow period in the observation well. This again suggests that both wells have opposing effects on the appearance of the late pseudo-radial flow regimes in the observation well and that what we observe is the resultant effect of these two opposing forces.

Chapter 6

Conclusion and Recommendation

6.1 Conclusion

The aim of this thesis was to extend the semi-analytical solution [17, 18, 19] for interference testing to take care of two parallel horizontal wells of unequal lengths and to use the modified model to study the effect of varying well lengths on flux distribution and pressure responses in both active and observation wells.

The result of this study is limited to the case in which the two horizontal wells are parallel and helps to improve our understanding of interference test analysis between a pair of parallel horizontal wells.

Some conclusions we have reached from this studies are:

1. Increasing the active well length for a fixed observation well length will decrease the active well pressure response and increase the observation well pressure response.
2. For a fixed active well length, increasing the observation well length has no significant influence on active well pressure response but will decrease the observation well pressure response.
3. Increasing the length of both wells simultaneously will cause a decline in the active well pressure response but will either increase or decrease the

observation well pressure response depending on the ratio of the well lengths and the well geometry.

4. For a fixed active well length increasing the observation well length has no significant influence on the flux distribution in the active well. However, the magnitude of flux in the observation well will increase.

5. Increasing the active well length for a given observation well length will

   a. neither delay nor hasten the appearance of the early radial flow period of the active well,

   b. neither delay nor hasten the outset of the intermediate linear flow period of the active,

   c. lengthen the intermediate linear flow period,

   d. delay the outset of the late pseudo-radial flow period of the active well, and

   e. hasten the outset of the late pseudo-radial flow period of the observation well.

6. Increasing the observation well length for a given active well length has no significant effect on any of the three flow regimes in the active well. However, it will delay the outset of the late pseudo-radial flow regime in the observation well pressure response.

6.2 Recommendation

We recommend the following for future research:

1. This model should be modified to incorporate the effects of wellbore storage.
2. This model should be generalized to handle the case in which the active and observation wells are not parallel.

Nomenclature

B : formation volume factor, rb/stb
c_t : total compressibility, psi^{-1}
f : fanning friction factor
h : reservoir thickness, ft
k : permeability, md
l : horizontal well half length, ft
L_h : horizontal well length, ft
N_{Re} : Reynolds number
P : pressure, psi
r_{we} : effective wellbore radius, ft
S : skin factor
t : time, hr
C_{hD} : dimensionless well conductivity
L_D : dimensionless well length
r_{wD} : dimensionless wellbore radius
q_D : dimensionless rate
P_D : dimensionless pressure
R_D : ratio of observation well length to active well length
t_D : dimensionless time
ε_D : dimensionless roughness factor
η_j : hydraulic diffusity in the direction x, y, or z
μ : viscosity, cp
ε : roughness factor, ft
ϕ : porosity
ρ : fluid density, lb/ft^3

Subscripts

w : wellbore
D : dimensionless
h : horizontal
i : initial
x : x direction
y : y direction
z : z direction

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